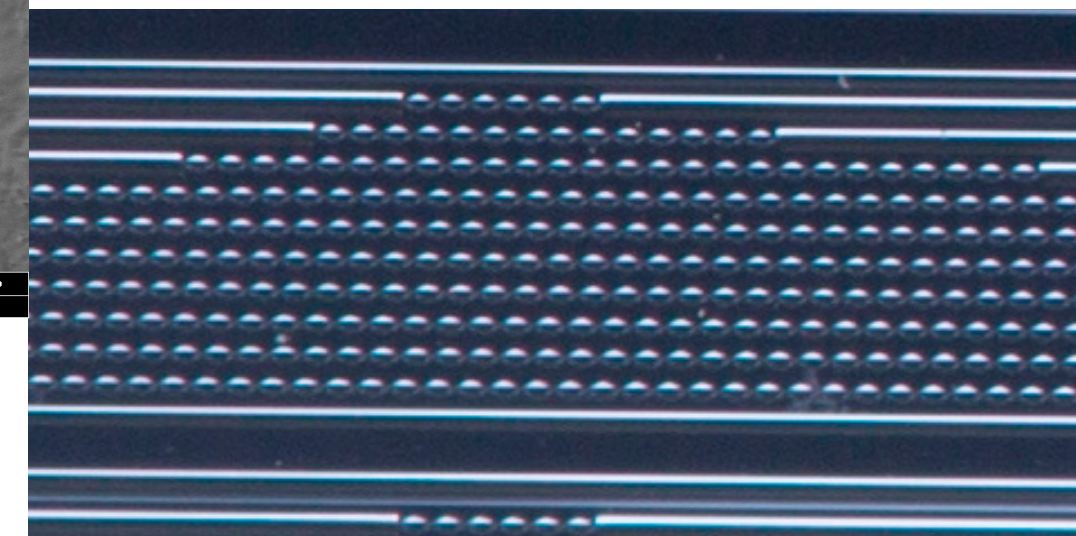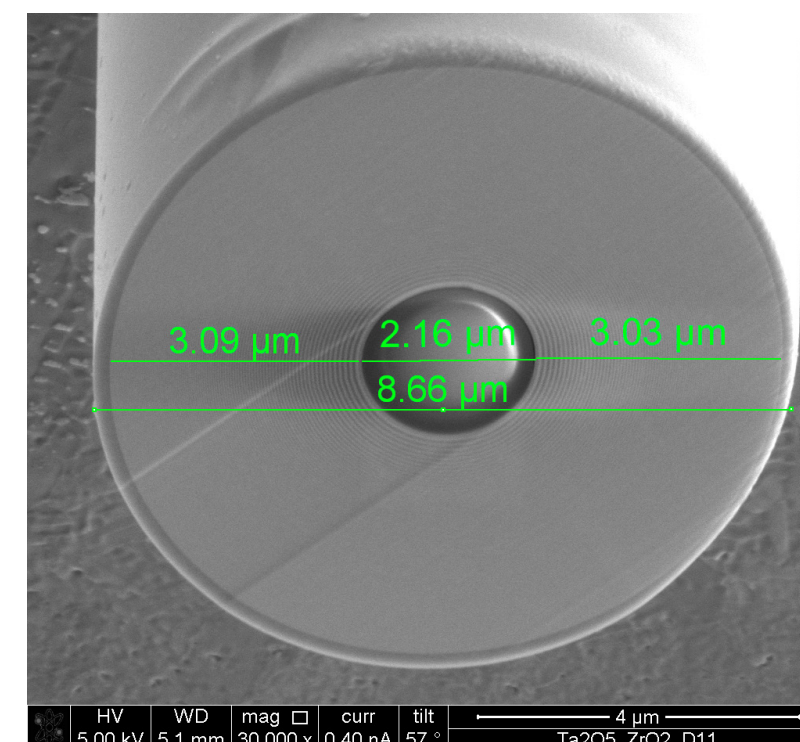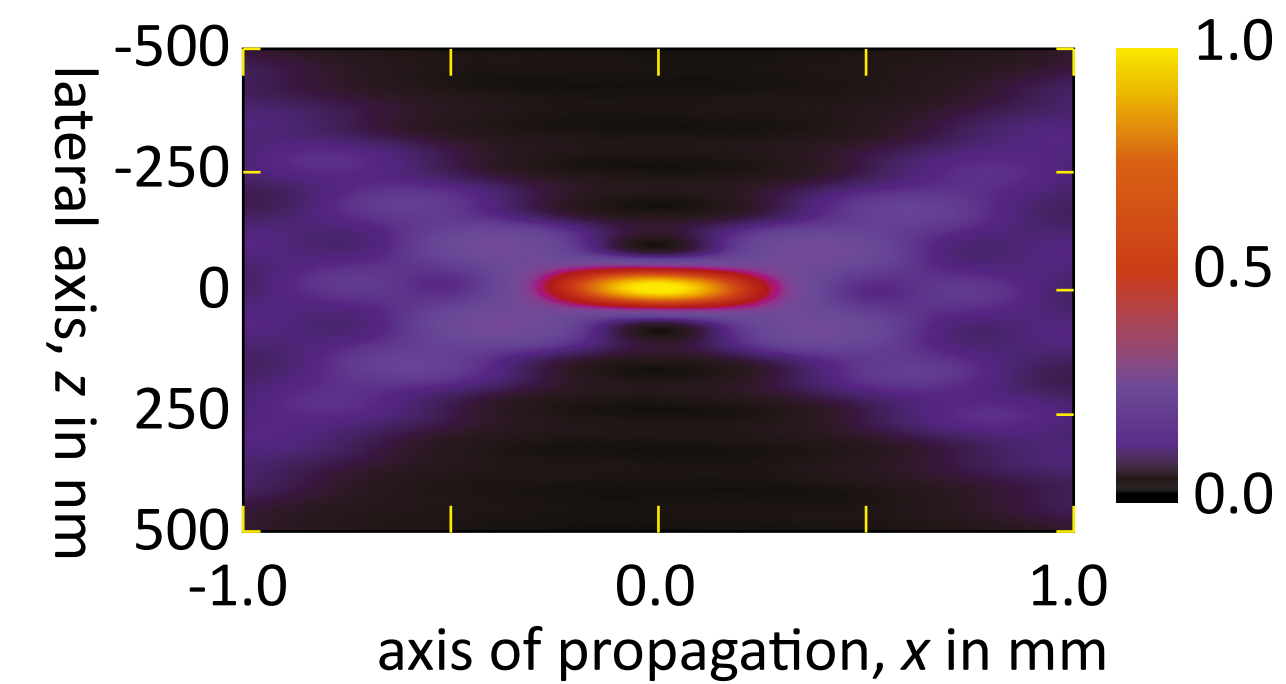# X-Ray Optics for Imaging

Hercules Specialised Course 19

Markus Osterhoff
University of Göttingen
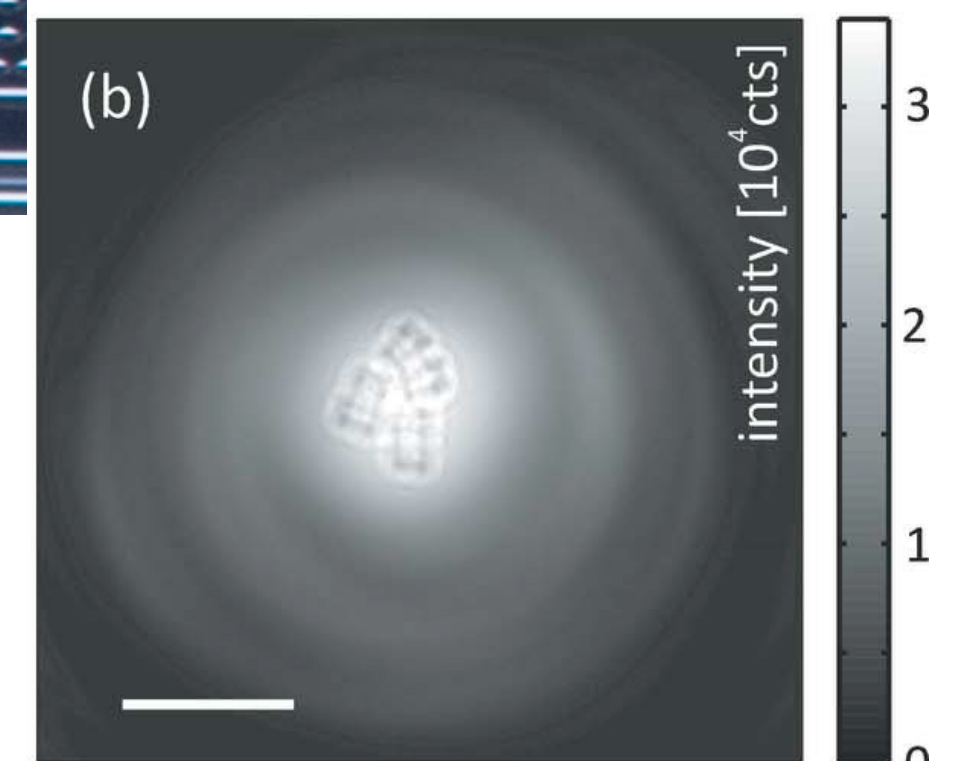
Grenoble, May 2017

ideal mirror

(a) Intensity distribution

Outline

► Introduction + History

► Reflective Optics

► Diffractive Optics

► Refractive Optics

► Waveguides

► Discussion

Advantages of Hard X-Rays

► high penetration length
looking into the volume,
3D tomography

► element specific
access to ionisation state
absorption, fluorescence, Auger

► short wavelength
matching length scales in crystals, glasses

► scattering from electron density

Disadvantages

► beam damage

► weak interaction with optics

► expensive

Outline

► *Introduction + History*

► Reflective Optics

► Diffractive Optics

► Refractive Optics

► Waveguides

► Discussion

What we want

► small focus,
i.e. high numerical aperture

► high efficiency

► long working distance,
i.e. high angles of convergence

► multiple contrasts

► weak interaction with sample;
low dose, but strong signal?

What we want

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  i.e. high numerical aperture

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► long working distance,
  i.e. high angles of convergence

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► weak interaction with sample;
  low dose, but strong signal?

What we have

► index of refraction
  $n = 1 - \delta + i\beta$

► δ: dispersive part,
  yields phase shift

► β: absorption

► typically: $\delta \sim 10^{-5}$, $\beta \sim 10^{-7}$ (hard x-rays)

► weak interaction with optics,
  x-rays cannot be bent so easily …

Not so much has happend …

► discovery of X-Rays
 by C.W. Röntgen in 1895

► no interaction with lenses found

► particles? waves? something else?

► what are crystals?
 arrays of atoms?

► lattice spacing + wavelength agree

William J. Morton, American Technical Book Co., 1896

Not so much has happend …

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by C.W. Röntgen in 1895

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▶ particles? waves? something else?

▶ what are crystals?
arrays of atoms?

▶ lattice spacing + wavelength agree

Generation of X-rays …

▶ only by traditional X-ray tubes

▶ parasitic use of synchrotron radiation

▶ dedicated synchrotron radiation sources

▶ ESRF as 3rd generation source

▶ 4th generation to come,
x-ray free electron lasers

▶ brilliance increase

Optics for (hard) X-rays:

▶ started about 50 years
after Röntgen's discovery
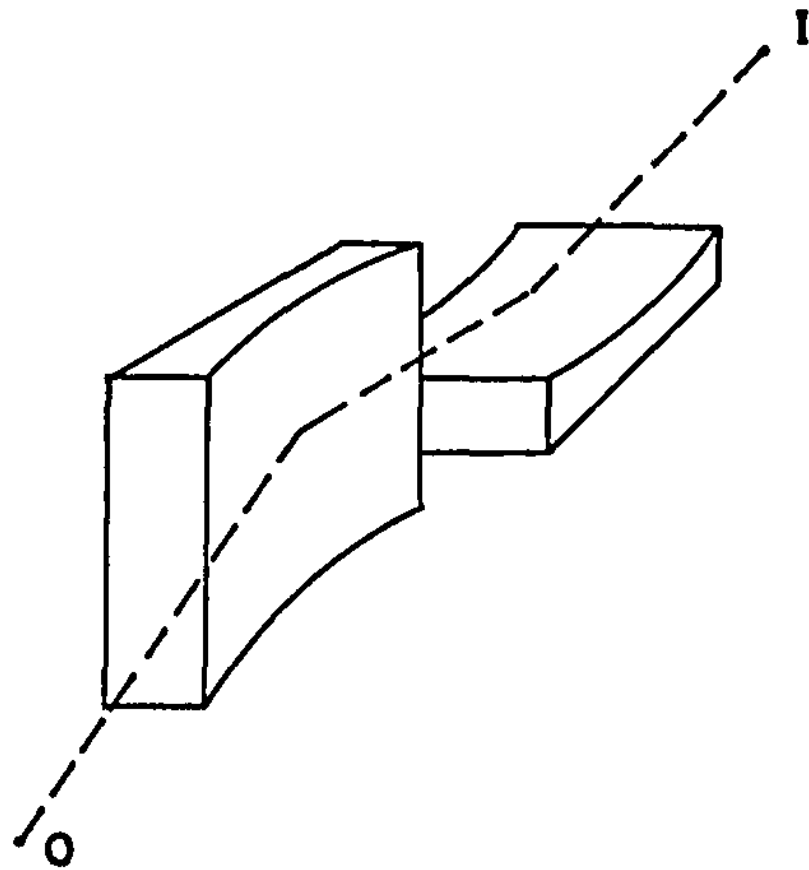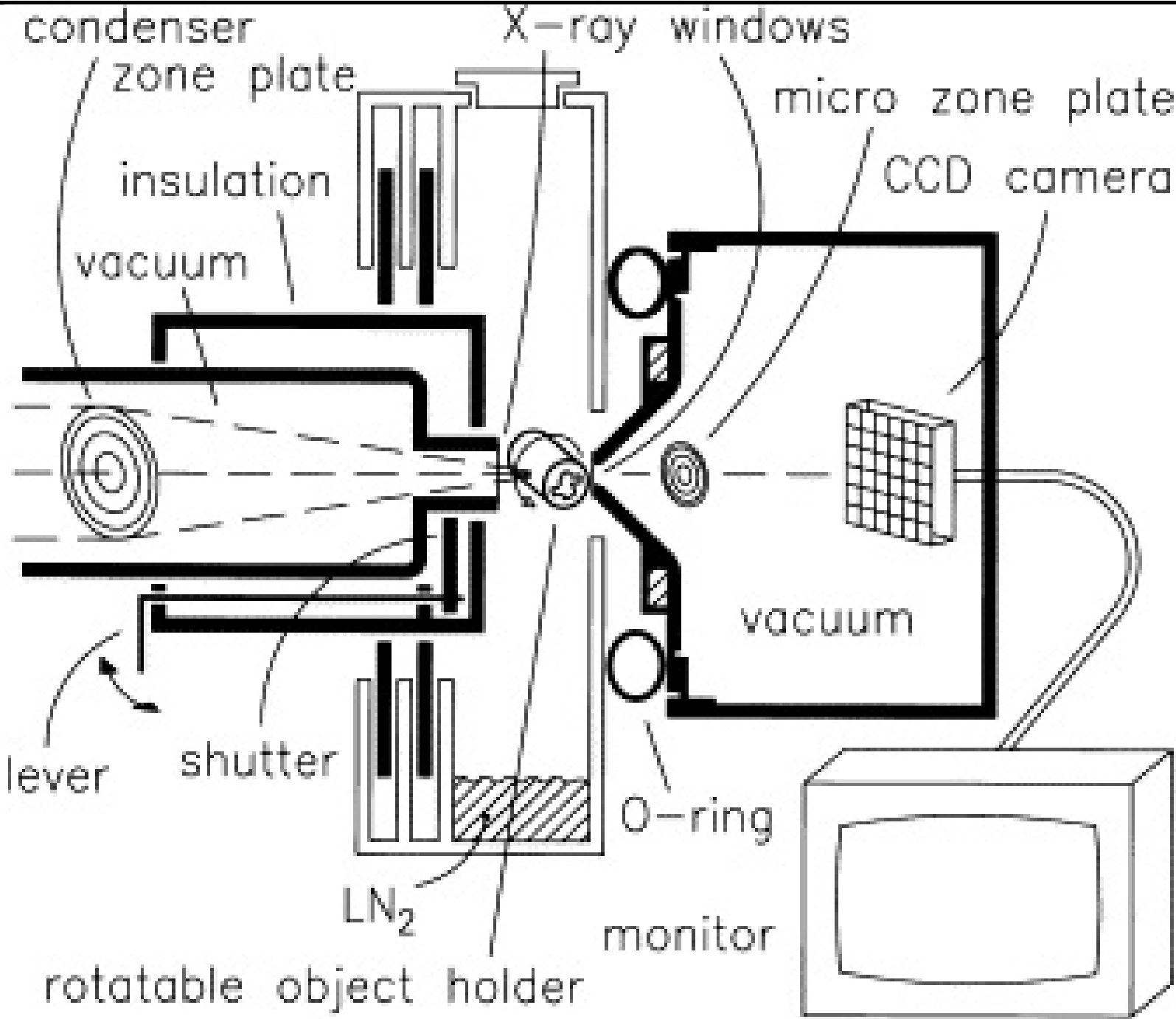
Pioneers of X-Ray Optics

FIG. 11. Arrangement of concave mirrors to produce real images of extended objects with incidence at small grazing angles.

THE development of techniques for focusing X-rays has occupied physicists for more than a century. Refractive lenses, which are used extensively in visible-light optics, are generally considered inappropriate for focusing X-rays, because refraction effects are extremely small and absorption is strong. This has lead to the development of alternative approaches[1,2] based on bent crystals and X-ray mirrors, Fresnel and Bragg–Fresnel zone plates, and capillary optics (Kumakhov lenses). Here we describe a simple procedure for fabricating refractive lenses that are effective for focusing of X-rays in the energy range 5–40 keV. The problems associated with absorption are minimized by fabricating the lenses from low-atomic-weight materials. Refraction of X-rays by one such lens is still extremely small, but a compound lens (consisting of tens or hundreds of individual lenses arranged in a linear array) can readily focus X-rays in one or two dimensions. We have fabricated a compound lens by drilling 30 closely spaced holes (each having a radius of 0.3 mm) in an aluminium block, and we demonstrate its effectiveness by focusing a 14-keV X-ray beam to a spot size of 8 µm.

The index of refraction for X-rays in matter can be written as $n = 1 - \delta + i\beta$, where β is the absorption index and δ is the

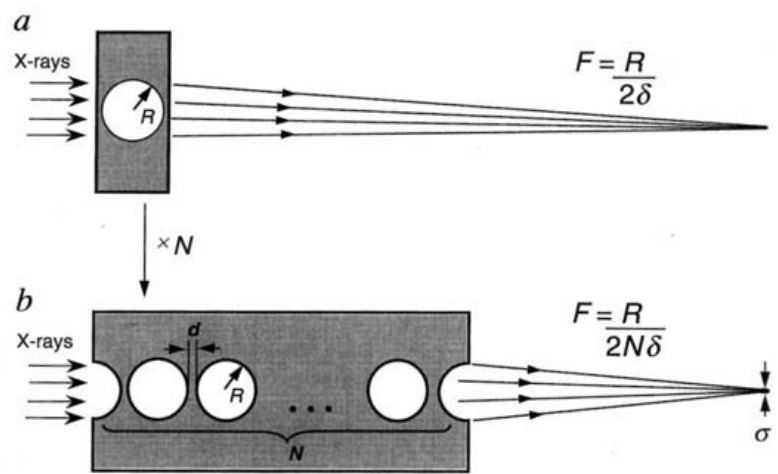‡ Present address: Physikalisches Institut, RWTH Aachen, 52056 Aachen, Germany.

FIG. 1 Schematic diagram showing the principles of X-ray focusing by a compound refractive lens (CRL). As $(1 - \delta)$ is smaller than 1 (where δ is the decrement of the refractive index), a collecting lens for X-rays must have a concave shape. a, A simple concave lens fabricated as a cylindrical hole in the material. b, A CRL consisting of a number (N) of cylindrical holes placed close together in a row along the optical axis, focuses the X-rays at a distance that is N times shorter compared to a single lens. R is the radius of the holes, d is the spacing between the holes, λ is the X-ray wavelength, and F is the focal distance for a parallel input beam.

refractive index decrement. Refraction being very small (δ is typically between 10^{-5} and 10^{-7}), all attempts to date to build refractive lenses for X-rays have been unsuccessful. Recently, the discussion about refractive lenses has been revived. Suehiro, Miyaji and Hayashi[3] have proposed a refractive lens of high-atomic-number (high-Z) material for focusing X-rays. Michette[4]

Kirkpatrick and Baez
1950ies: reflecting mirrors

G. Schmahl and coworkers
1980ies: Fresnel Zone Plates

A. Snigirev and his team
1990ies: Refractive Lenses
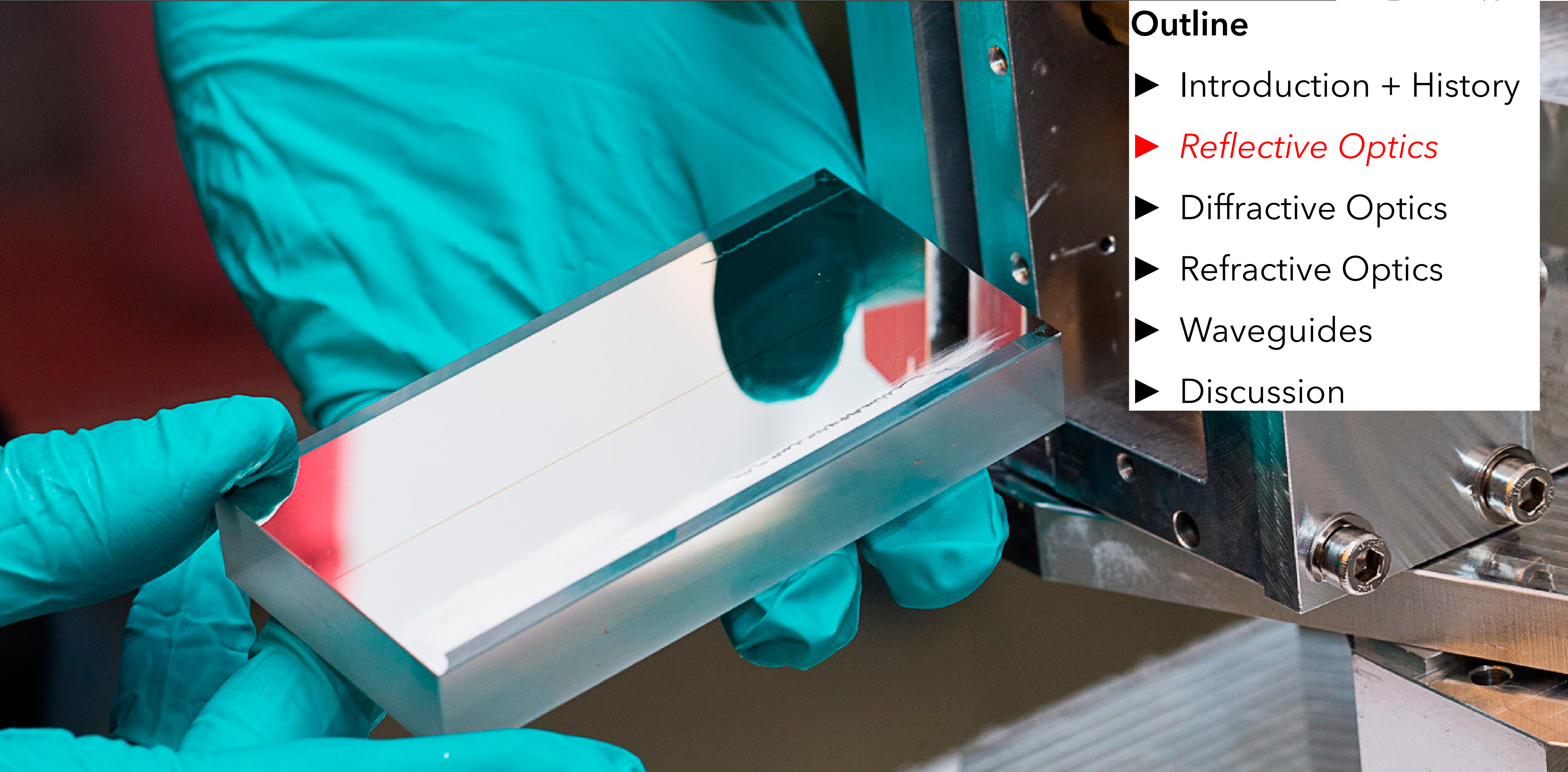
Kirkpatrick, Baez, JOSA 1948
Schmahl et al, Ultramicroscopy, 2000
Snigirev, Lengeler et al, Nature, 1996
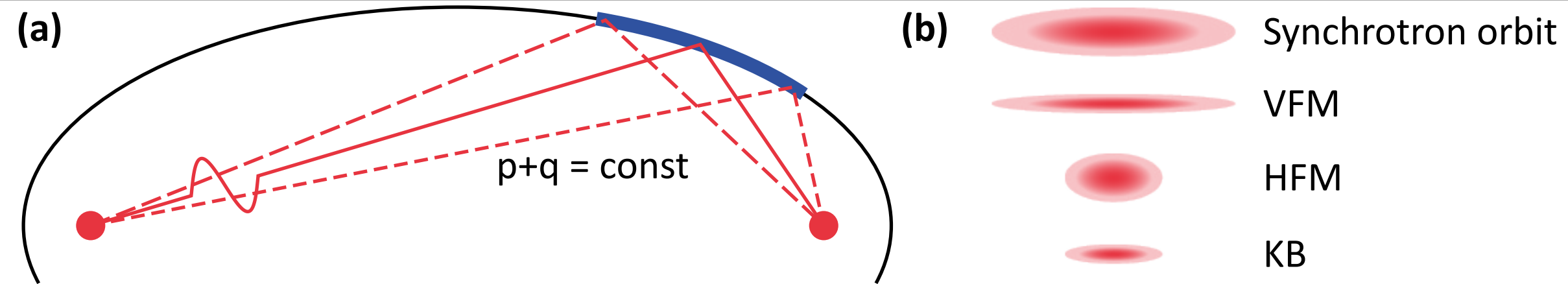
Mirrors and Multilayer Mirrors

(a)

p+q = const

(b)

Synchrotron orbit

VFM

HFM

KB

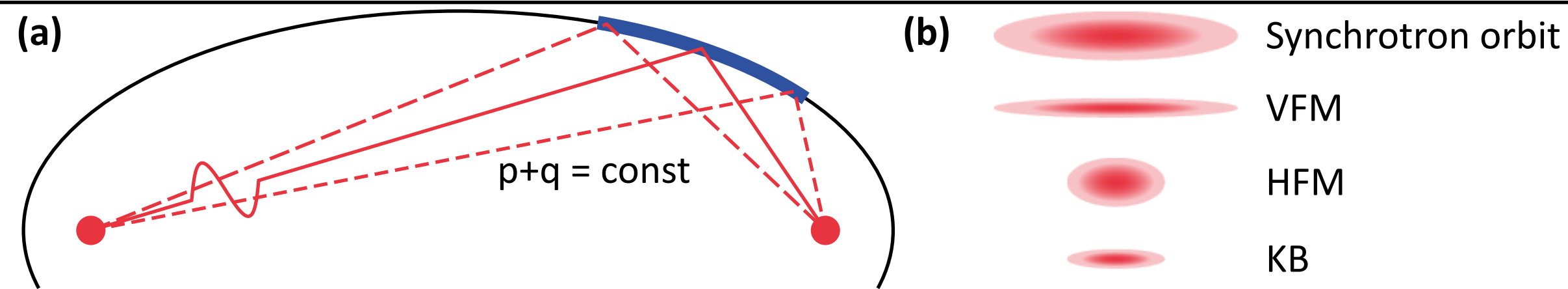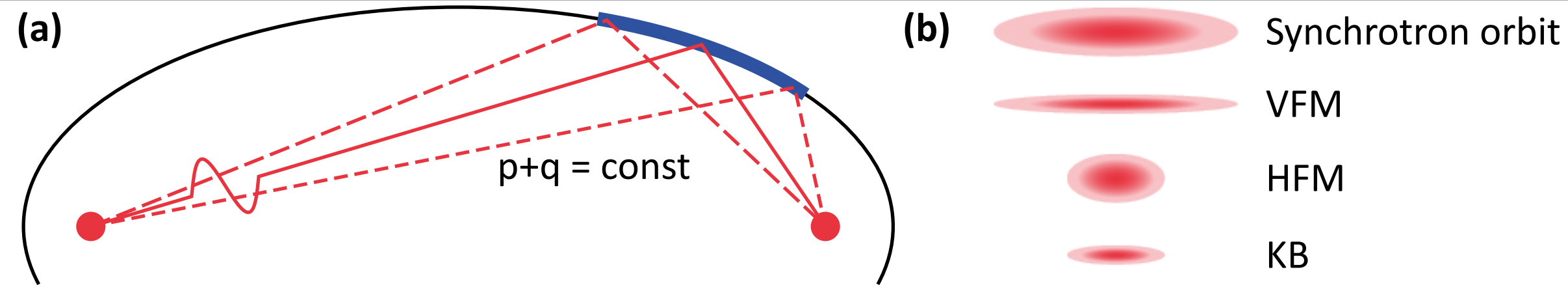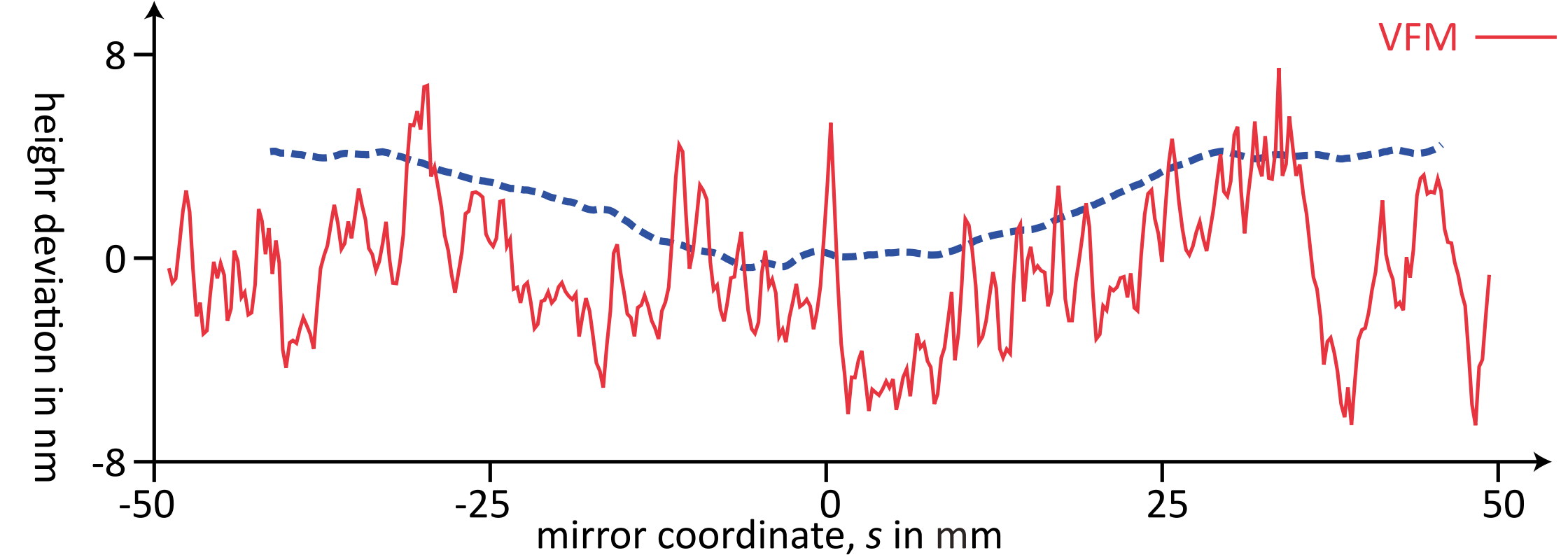## Ellipse:

► the distance from S to F via any point on the ellipse is constant

► constant distance = constant phase

► waves interfere constructively, leading to a focus

(a)

p+q = const

(b)

Synchrotron orbit

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Ellipse:

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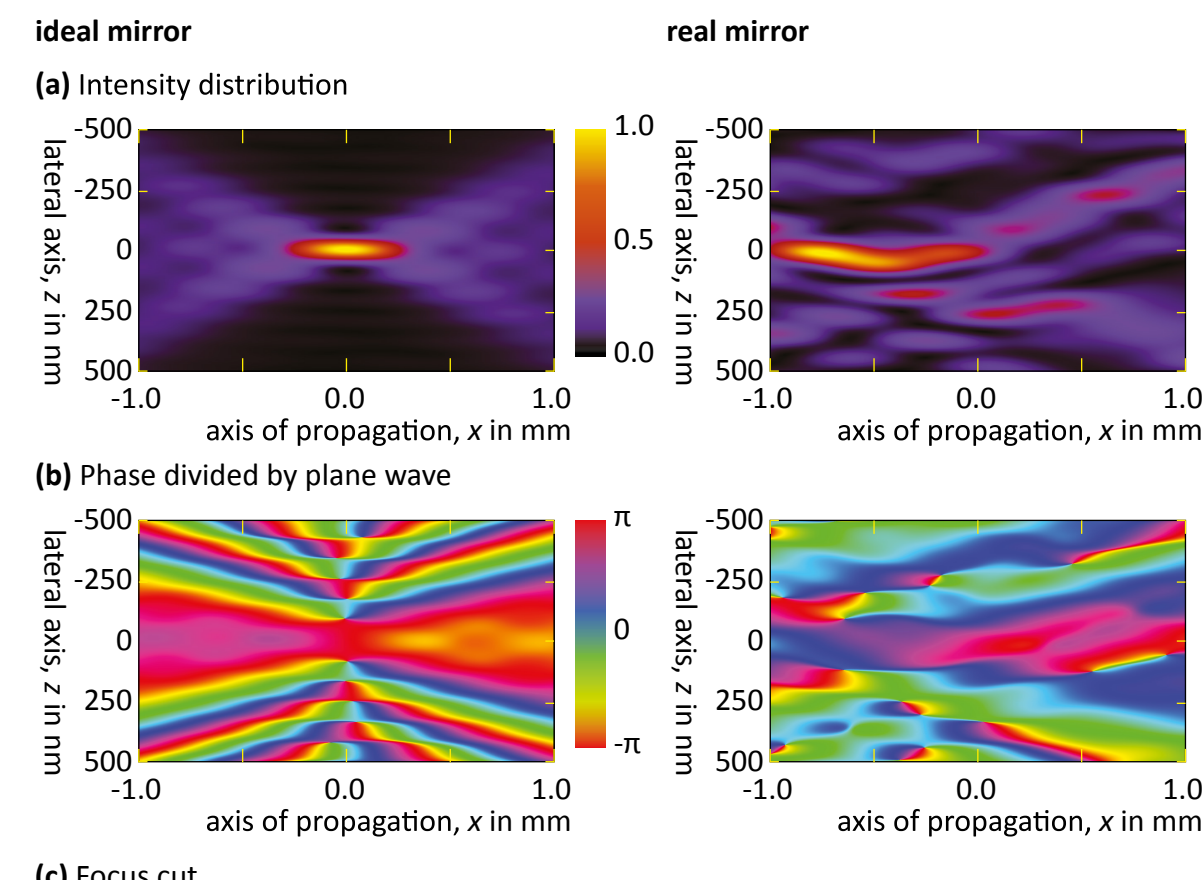
► waves interfere constructively, leading to a focus

Requirements:

► high reflectivity of the surface
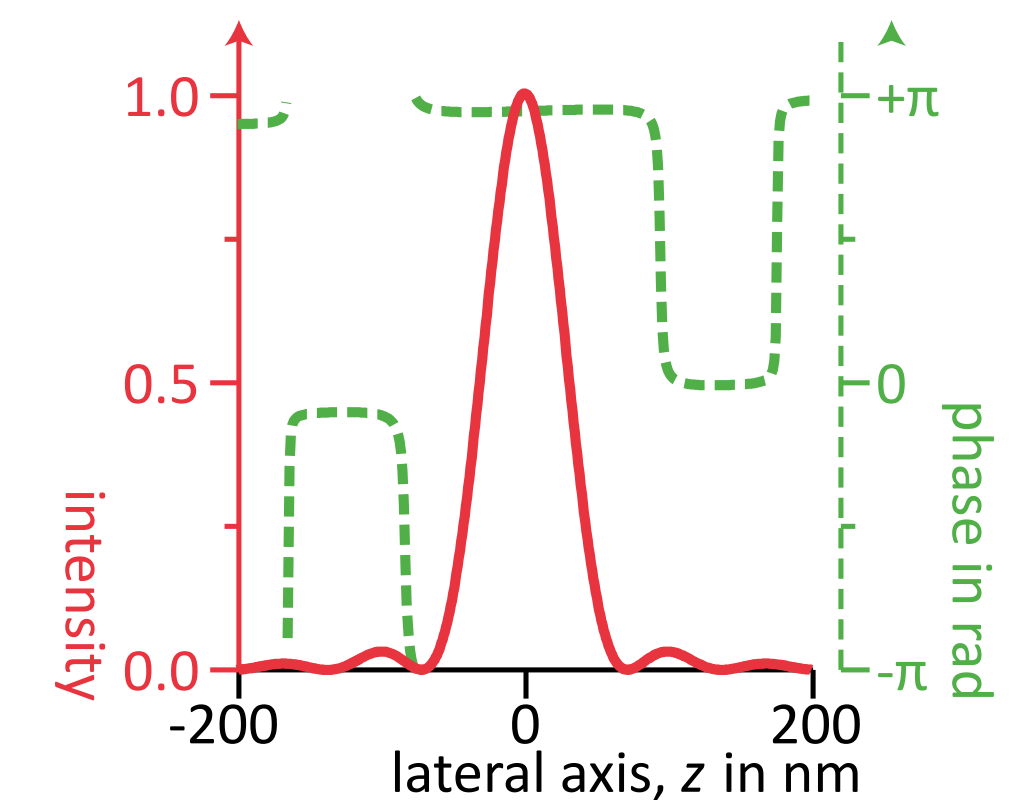
► good surface quality

► 3D shape or 2 crossed 2D shapes

Principle and Geometry

(a)

p+q = const

(b)

Synchrotron orbit

VFM

HFM

KB

Ellipse:

▶ the distance from S to F via any point on the ellipse is constant

▶ constant distance = constant phase

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Requirements:

▶ high reflectivity of the surface

▶ good surface quality

▶ 3D shape or 2 crossed 2D shapes

Measured height deviation profiles

HVM ----
VFM ——

typical figure errors, 2010: ~ nm over 10 cm

ideal mirror   **real mirror**   **(c)** Focus cut

(a) Intensity distribution

(b) Phase divided by plane wave

(c) Focus cut

wave-optical simulations of an ideal and real mirror; focal spot size <100 nm (point-source)
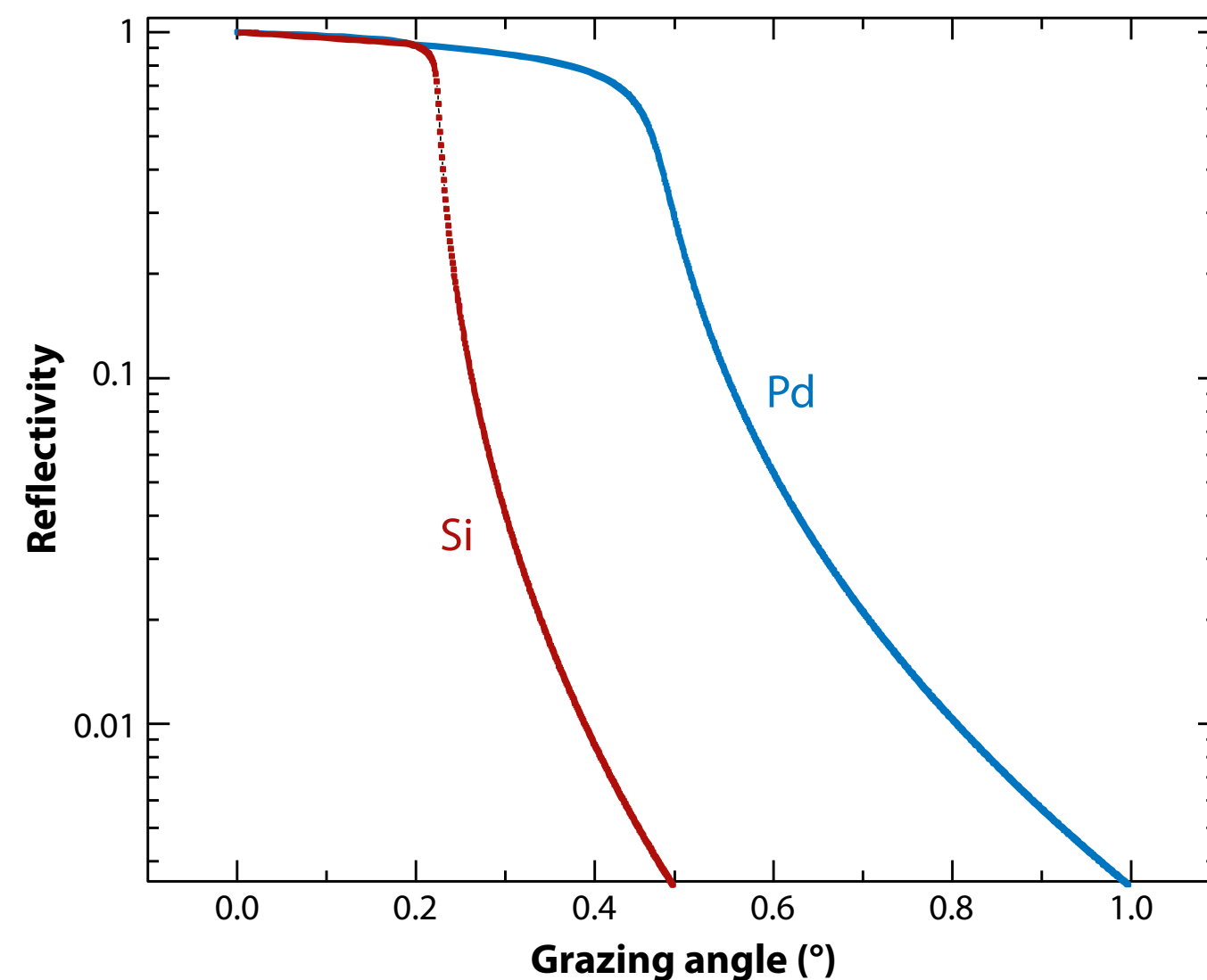
Osterhoff, PhD thesis, 2011
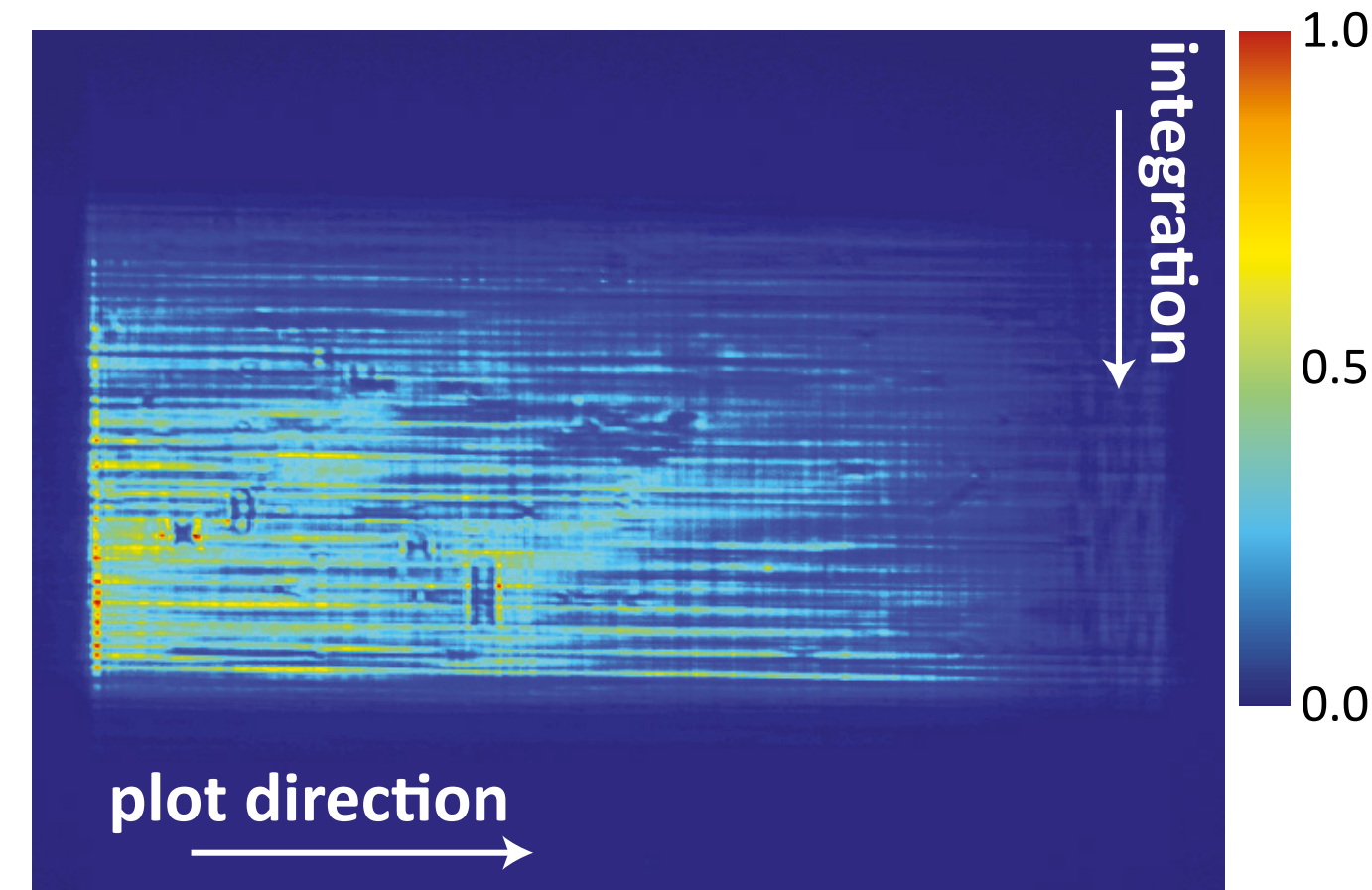
3. X-RAY MIRRORS

X-ray mirrors rely on total external reflection. The real part of the index of refraction is given by (6, 7)

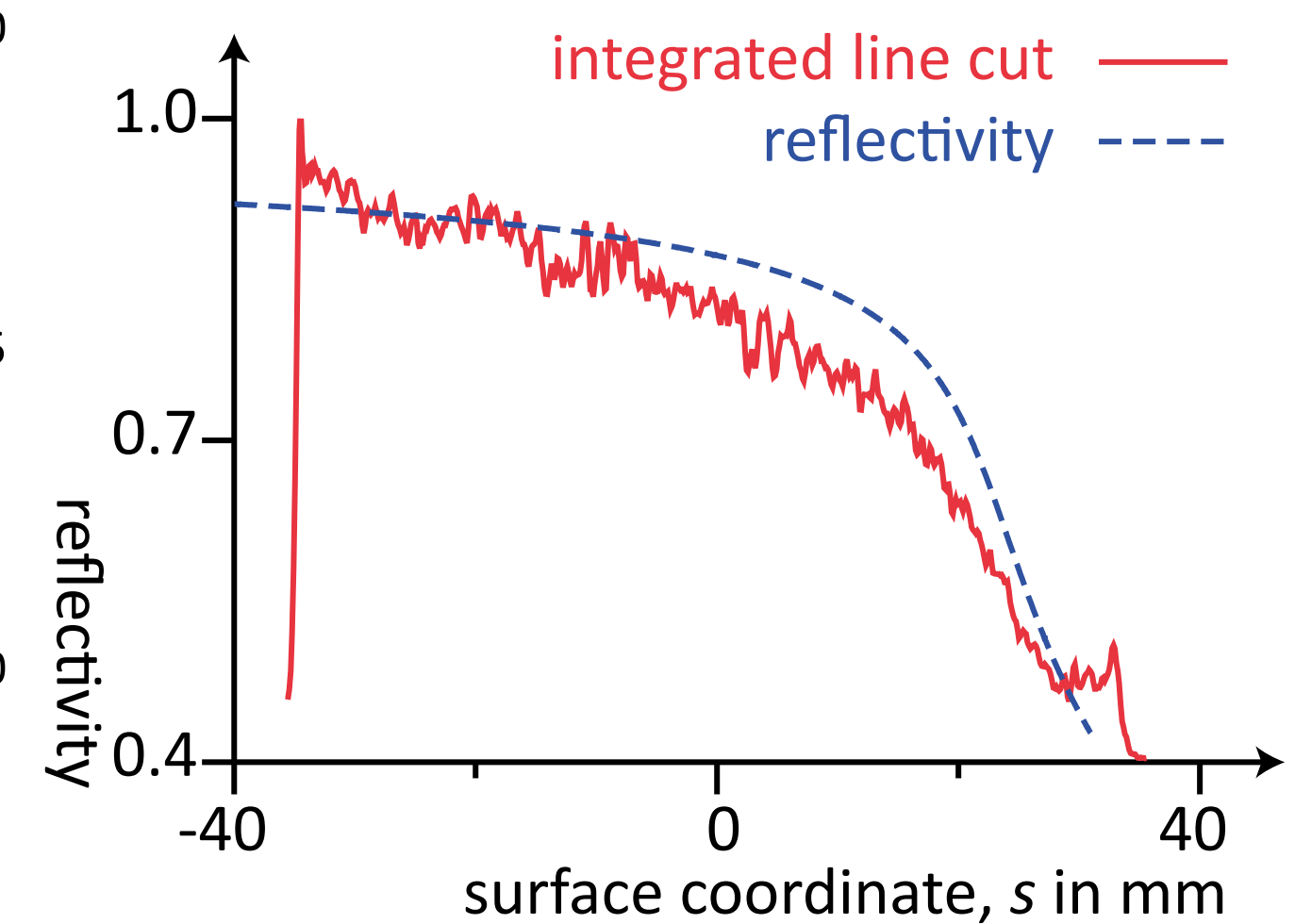$$n = 1 - \delta = 1 - \lambda^2 r_e \rho / 2\pi. \qquad 2.$$

Here λ is the wavelength; r_e is the classical electron radius, which equals 2.814×10^{-5}; and ρ is the electron density. The result is that n is less than unity so that, according to Snell's law, there is total external reflection, and because δ is very small ($\sim 10^{-5}$), reflection occurs only at low grazing angles. The reflectivity of smooth surfaces can be computed from Fresnel formulas (8). The critical grazing angle above which total external reflection does not occur is given by $\theta_c = \sqrt{(2\delta)}$. **Figure 5** shows calculated (9) specular reflectivity curves at 8 keV for a Si mirror without any coating as well as for one coated with Pd. At extremely grazing angles, an evanescent wave to a depth of $\lambda/(2\pi\theta_c)$ propagates just below the surface. The thickness of the coating should be chosen to be much larger than the penetration depth of the evanescent wave, which is 3 nm in the case of a Pd mirror.
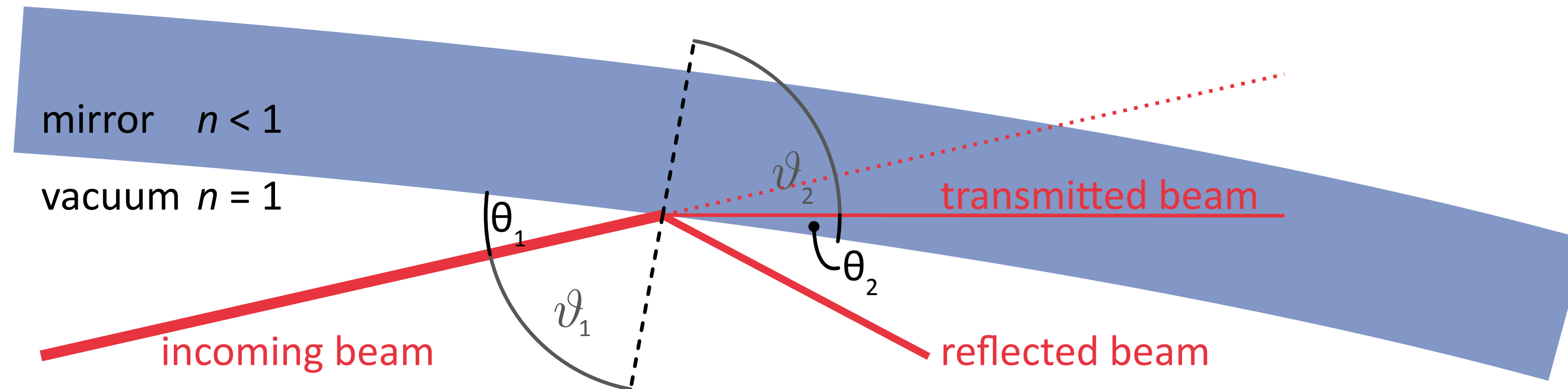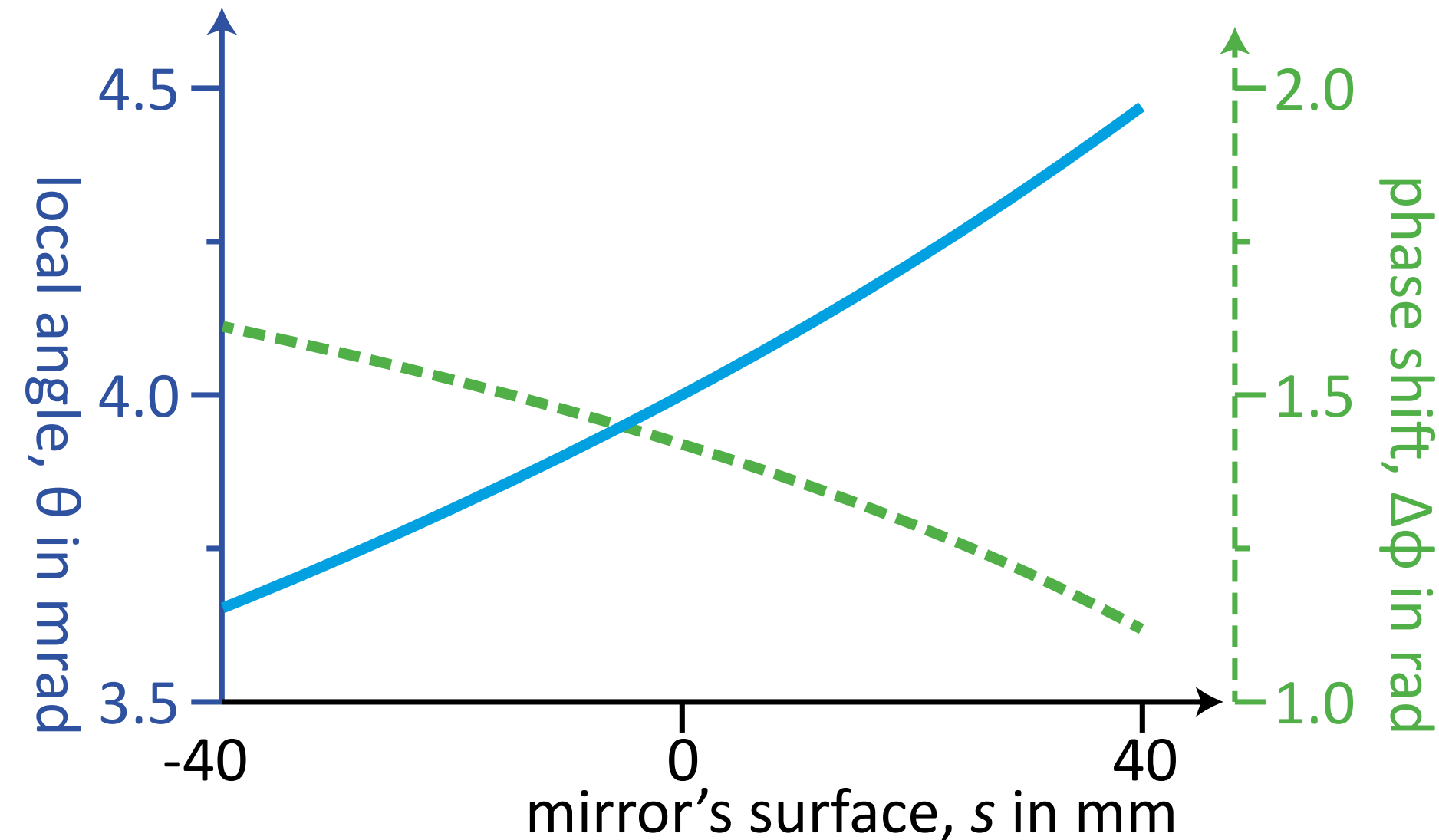
(a) KB mirror far-field

(b) Fresnel reflectivity

▶ total external reflection for grazing incidence

▶ typical angles: 0.5° – few milli radian

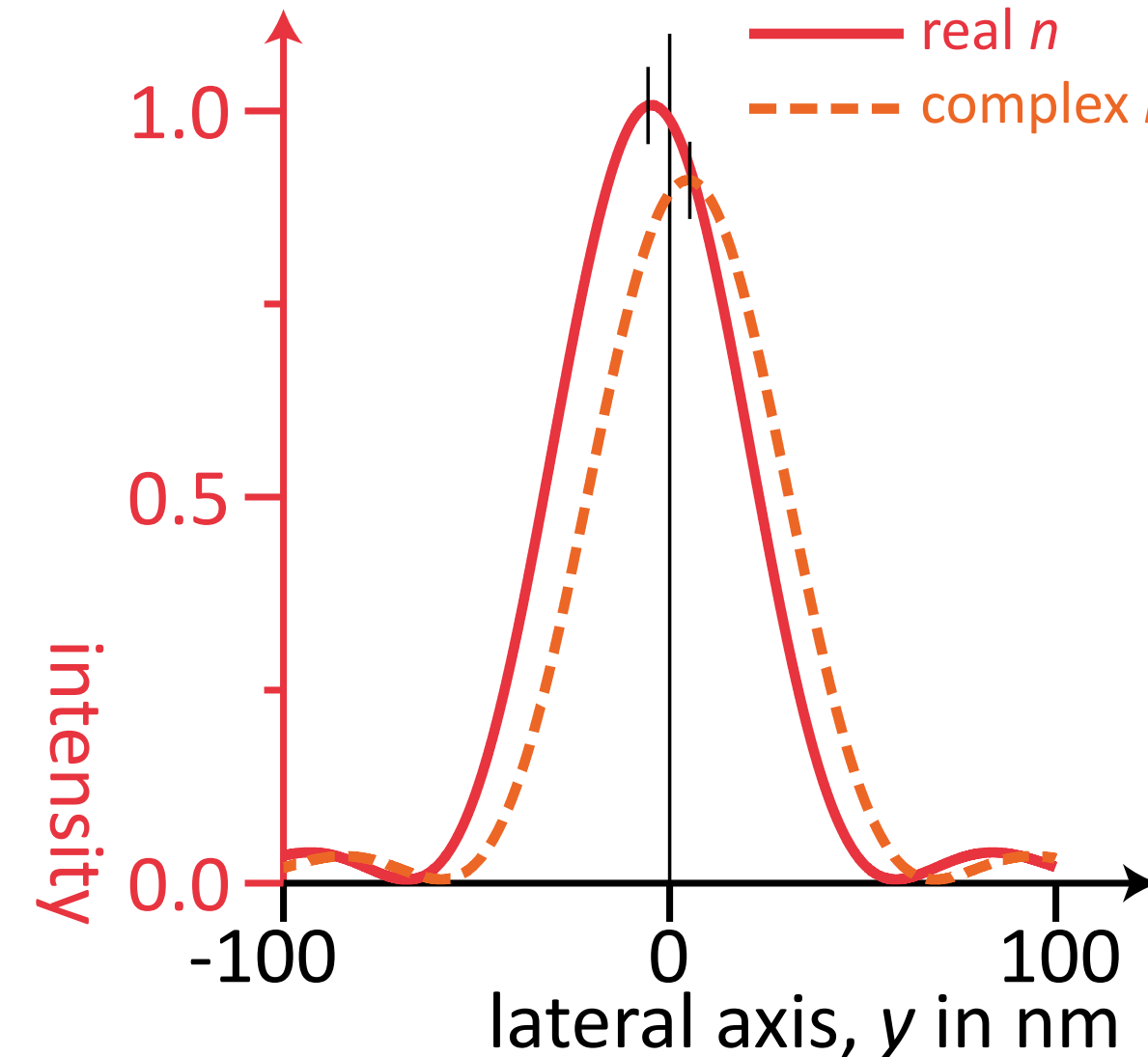▶ proportional to electron density × wavelength

Gouy Phase

$$\phi(z_m) = 2 \arctan\left(\frac{-\sqrt{2\delta - \sin^2 \theta_z}}{\sin \theta_z}\right)$$

mirror $n < 1$

vacuum $n = 1$
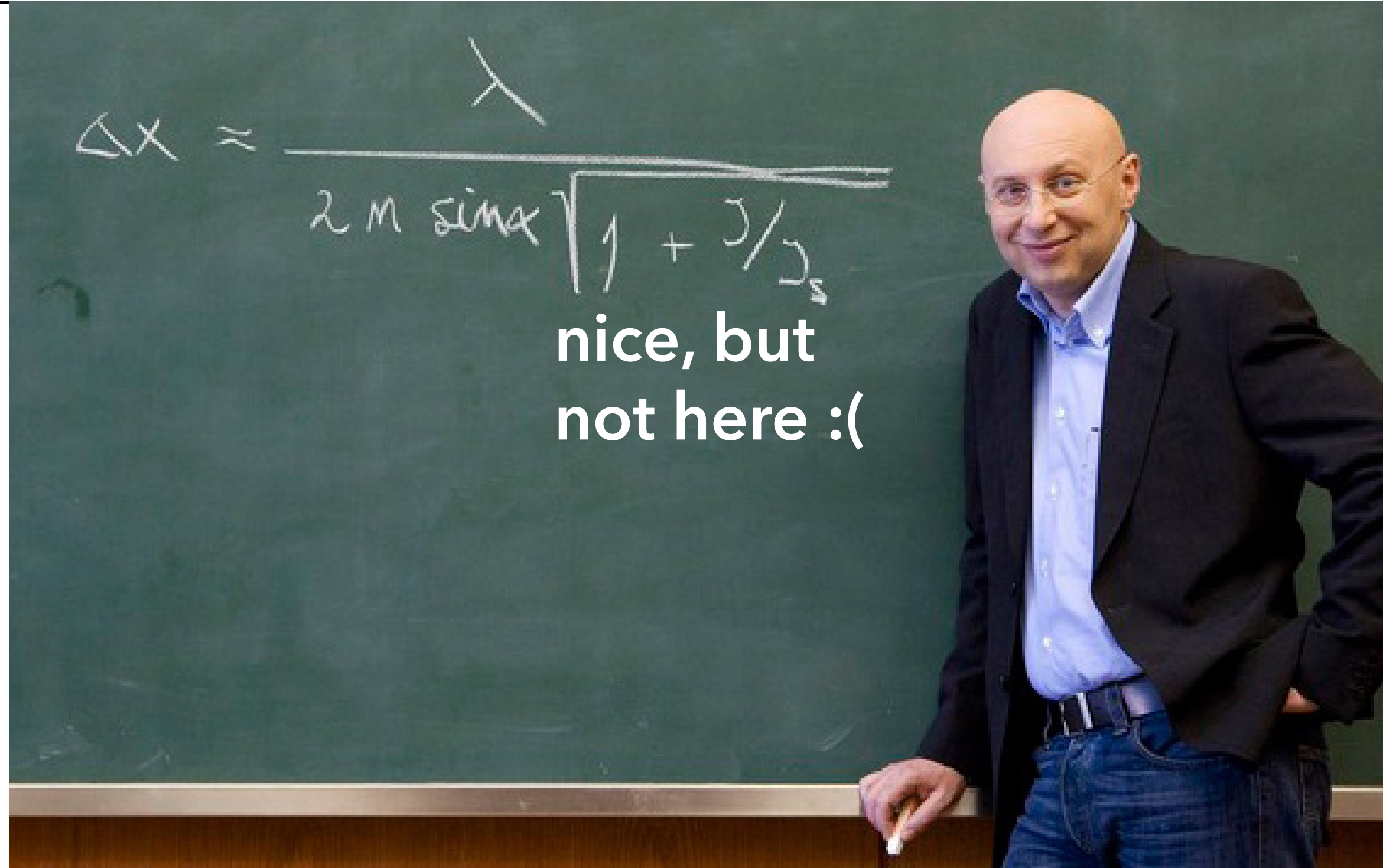
θ_1

ϑ_1

ϑ_2

transmitted beam

θ_2

incoming beam

reflected beam

(a) Local angle and phase shift

local angle, θ in mrad

phase shift, $\Delta\phi$ in rad

mirror's surface, s in mm

(b) Simulated focus

real n

complex n

intensity

lateral axis, y in nm

► evanescent wave enters even under total reflection

► surface: phase shift gradient

► shift of focus position

► reduced intensity due to absorption

► but no impact on focus size, no aberrations

Kewish et al, Applied Optics, 2007
Osterhoff, PhD thesis, 2011

$$\Delta x \approx \frac{\lambda}{2\, n\, \sin\alpha \sqrt{1 + \frac{\lambda}{2\pi}}}$$

nice, but
not here :(

In the TRM case one assumes a full opening of up to half of the critical angle
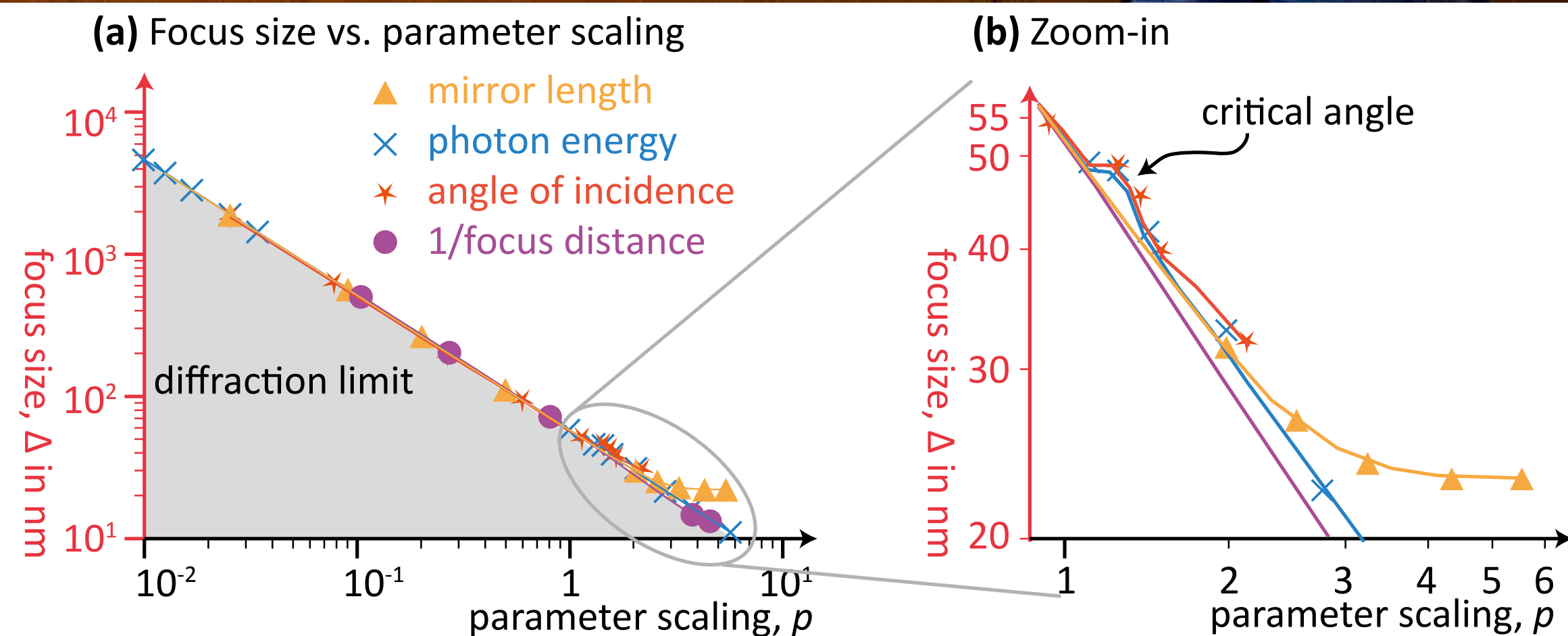
$$4\varepsilon \le \theta_C \tag{7}$$

leading to a diffraction limit of

$$D_{diff}(TRM) \approx \frac{1.76\,\lambda}{\sqrt{2\,\delta}} = 1.76\sqrt{\frac{\pi}{r_0\,\rho_e}} \tag{8}$$

Harke, Hell et al, Optics Express, 2008 Morawe, Osterhoff, NIMA, 2010

$$\Delta x \approx \frac{\lambda}{2\, m\, \sin\alpha \sqrt{1 + \nu/\nu_s}}$$

nice, but not here :(

In the TRM case one assumes a full opening of up to half of the critical angle

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leading to a diffraction limit of

$$D_{diff}(TRM) \approx \frac{1.76\,\lambda}{\sqrt{2\,\delta}} = 1.76\sqrt{\frac{\pi}{r_0\,\rho_e}} \tag{8}$$

The beam size limit $\sim W_c$, derived here for waveguiding geometries, also applies to other x-ray focusing devices. For example, the spot size achievable with a Fresnel

► index of refraction scales as λ^2

► numerical aperture vanishes

► fundamental focus limit (!/?)

(a) Focus size vs. parameter scaling

- ▲ mirror length
- ✕ photon energy
- ✱ angle of incidence
- ● 1/focus distance

focus size, Δ in nm

diffraction limit

parameter scaling, p

(b) Zoom-in

critical angle

focus size, Δ in nm

parameter scaling, p

Harke, Hell et al, Optics Express, 2008   Morawe, Osterhoff, NIMA, 2010
Osterhoff, PhD thesis, 2011   Bergemann et al, PRL, 2003

Multilayer Coatings, Takagi-Taupin

(a) Confocal ellipses

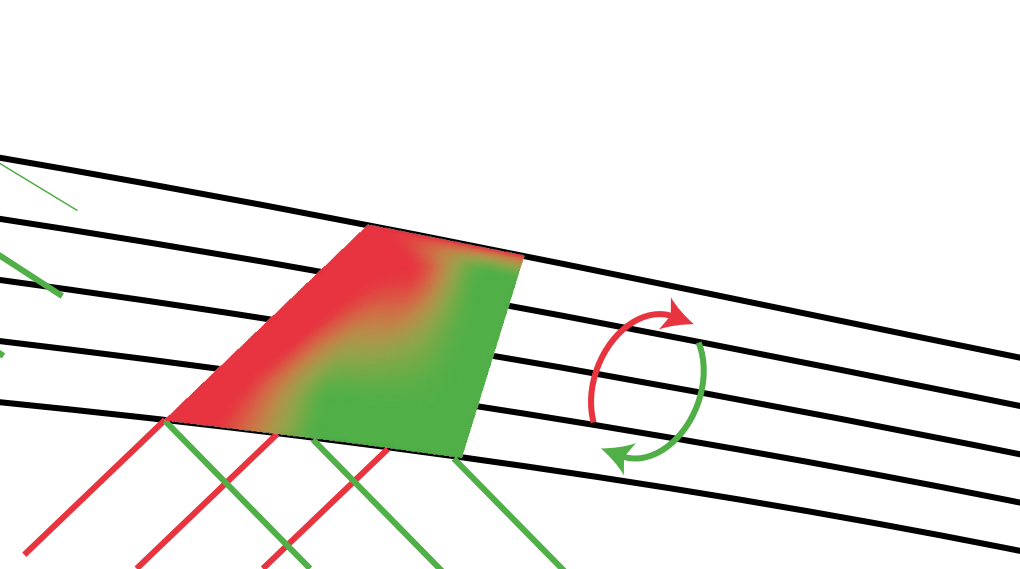**(b)** Elliptical coordinates

ML mirror as nested confocal ellipses

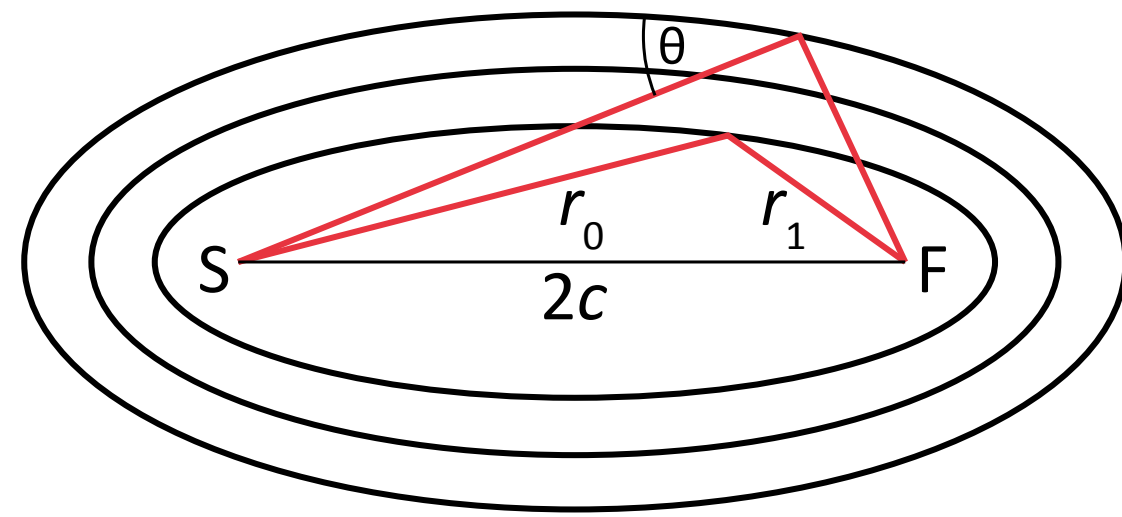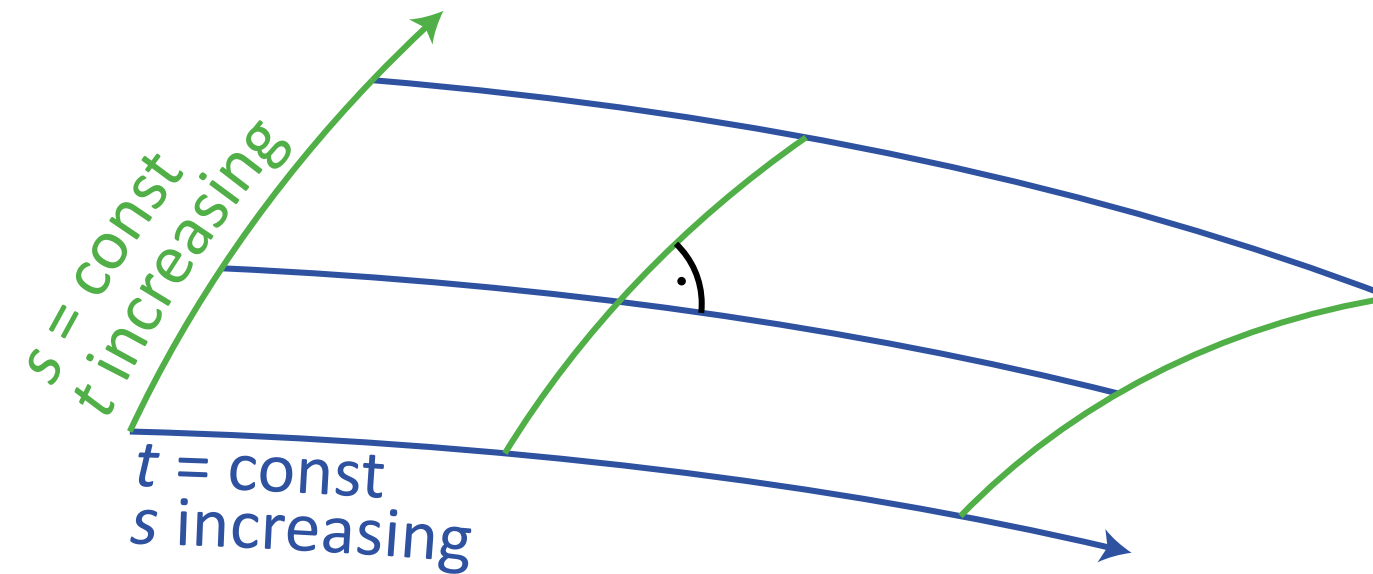**(a)** Kinematical theory   **(b)** Ray-tracing   **(c)** Dynamical theory

Multilayer Coatings, Takagi-Taupin
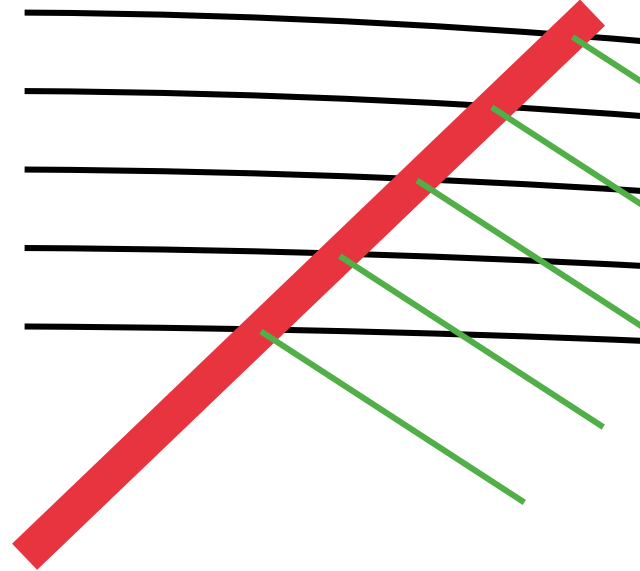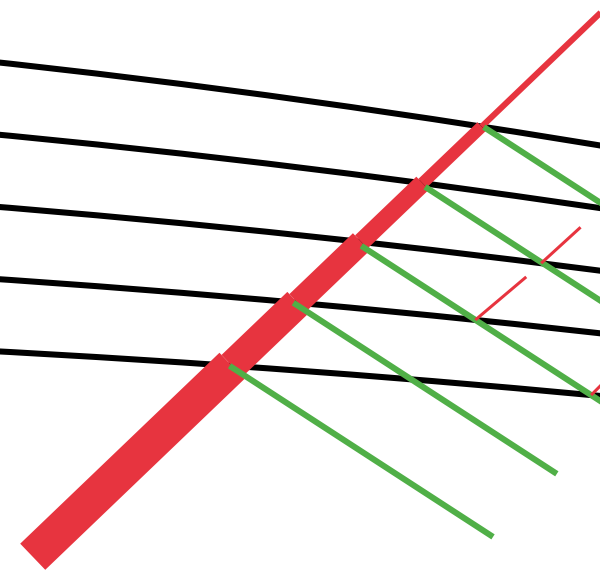
(a) Confocal ellipses

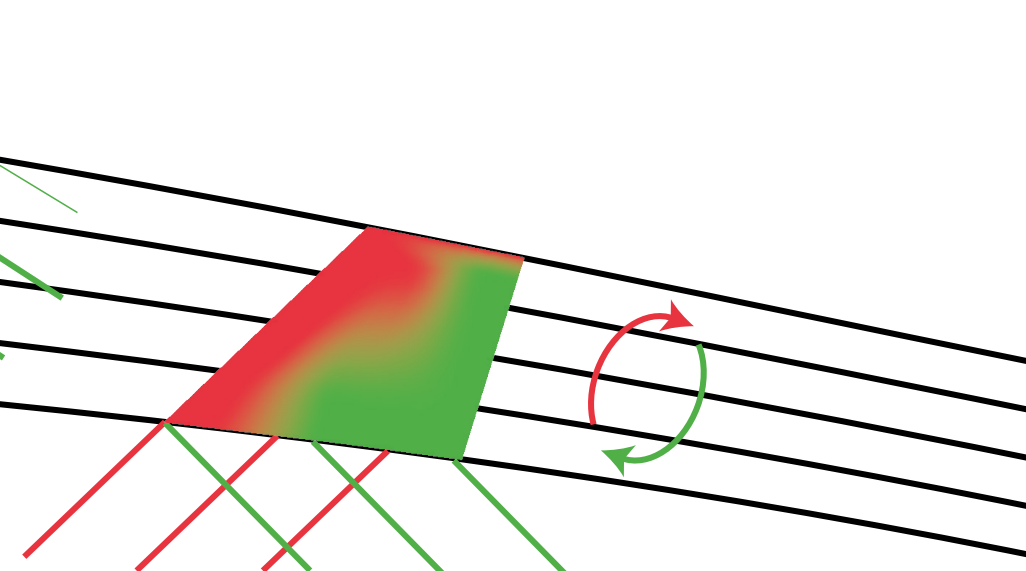**(b)** Elliptical coordinates

ML mirror as nested confocal ellipses

(a) Kinematical theory   **(b)** Ray-tracing   **(c)** Dynamical theory

models of increasing complexity: single reflection;
multiple reflection; coupled wave-theory

$$(\alpha^2 \partial_s + \beta^2 \partial_t)\psi_0 = i(u_0\psi_0 + u_1\psi_1) - \gamma^+ \psi_0$$

$$(\alpha'^2 \partial_s - \beta'^2 \partial_t)\psi_1 = i\Big((u_0 - \varepsilon)\psi_1 + u_{\mathsf{T}}\psi_0\Big) + \gamma'^- \psi_1$$

Takagi-Taupin description in elliptical geometry:
incoming wave and reflected wave
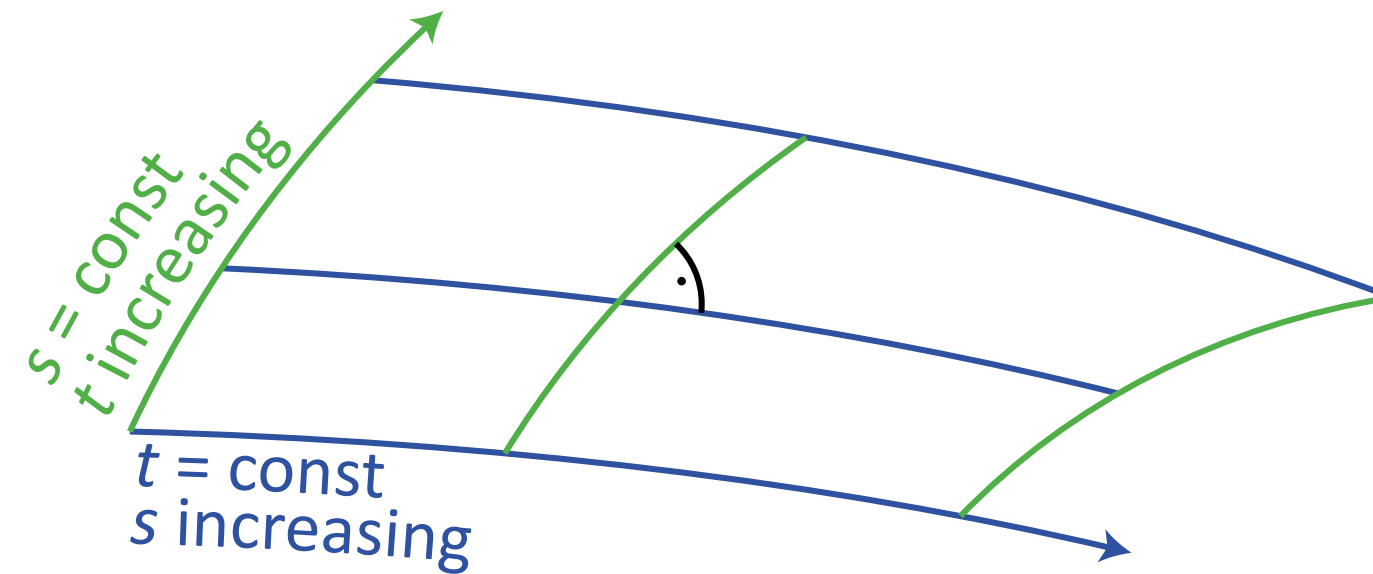are coupled by quasi-periodic susceptibility

Multilayer Coatings, Takagi-Taupin
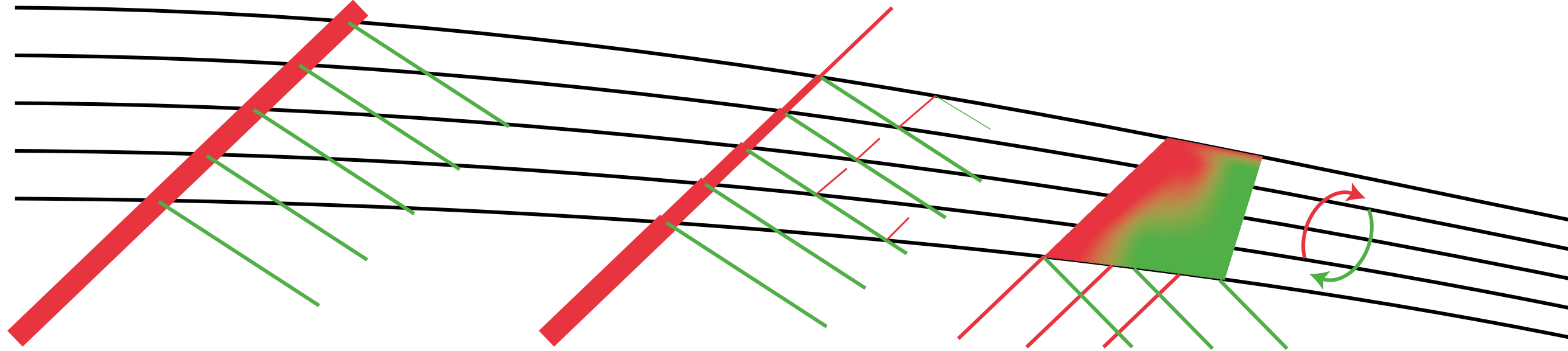
(a) Confocal ellipses

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ML mirror as nested confocal ellipses
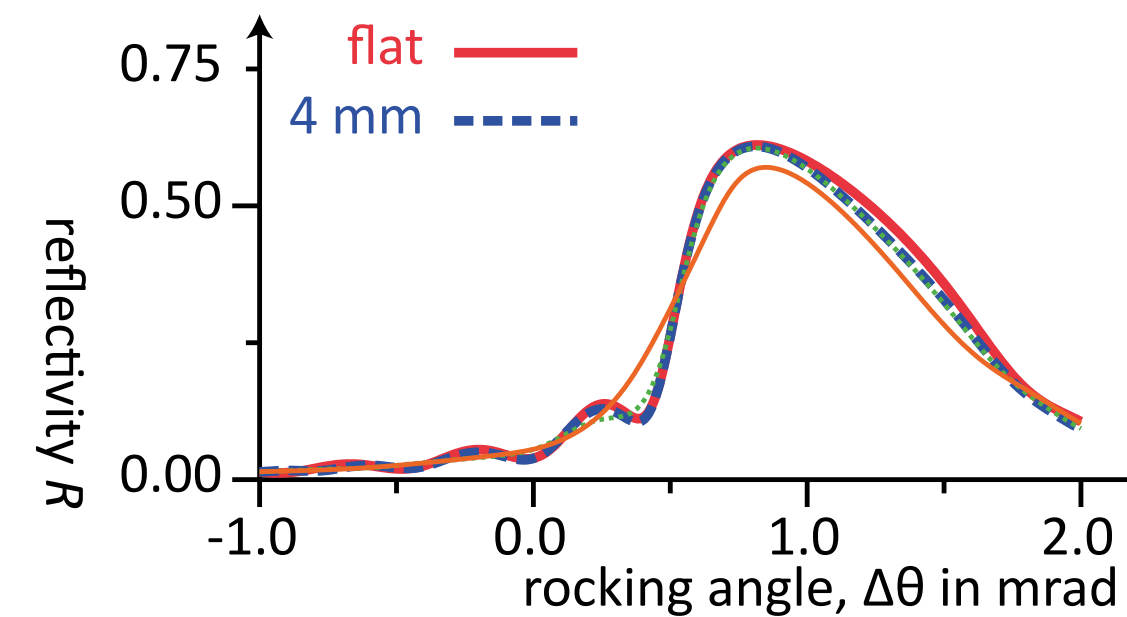
(a) Kinematical theory   **(b)** Ray-tracing   **(c)** Dynamical theory
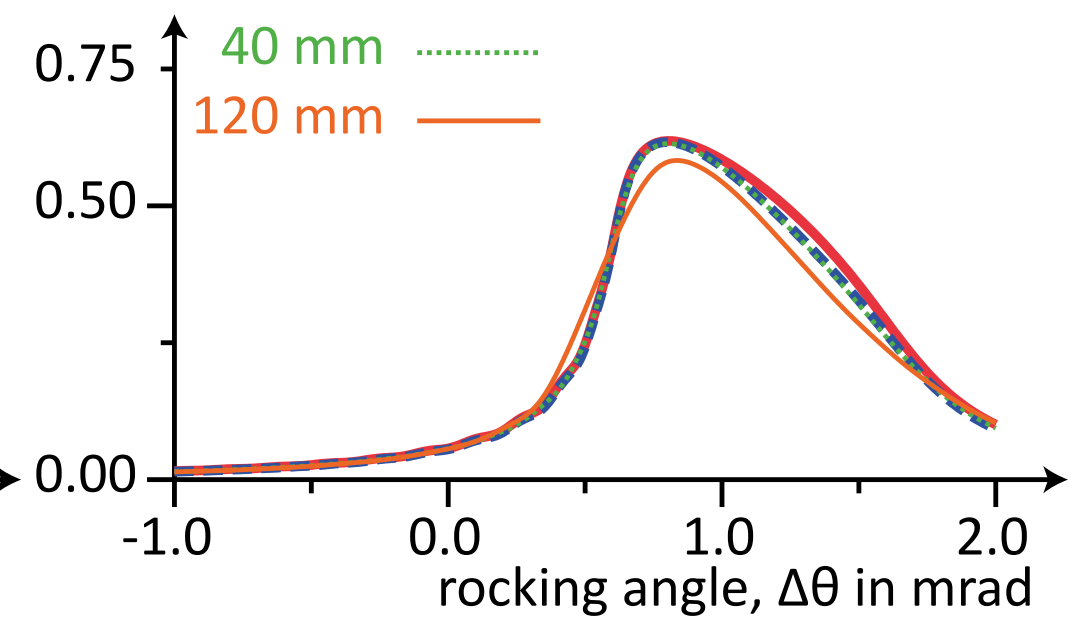
models of increasing complexity: single reflection;
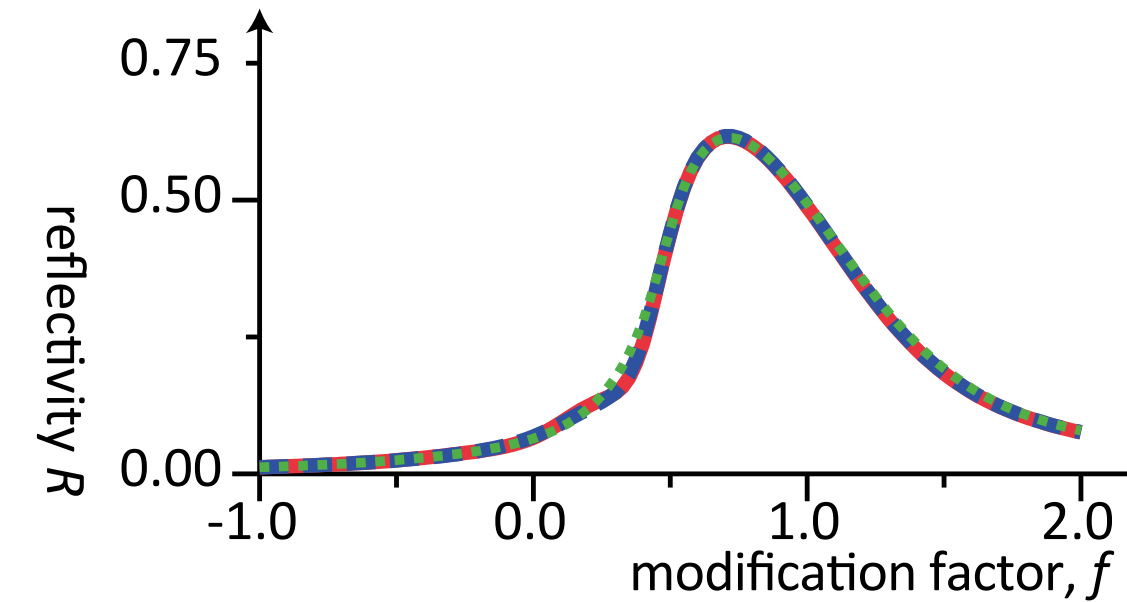multiple reflection; coupled wave-theory
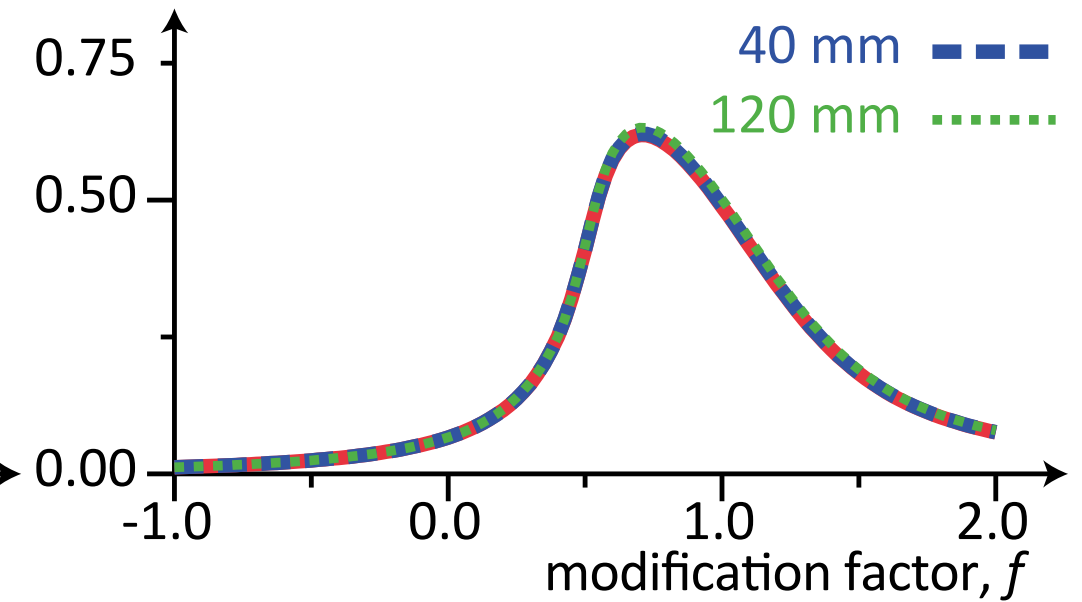
(a) 12.4 keV, 20 layers

(b) 12.4 keV, 50 layers

(a) 12.4 keV, 20 layers

(b) 12.4 keV, 50 layers

simulated reflectivity of curved ML mirror;
top: without, bottom: with
correction for refraction inside ML

$$(\alpha^2 \partial_s + \beta^2 \partial_t)\psi_0 = i(u_0\psi_0 + u_1\psi_1) - \gamma^+ \psi_0$$

$$(\alpha'^2 \partial_s - \beta'^2 \partial_t)\psi_1 = i\Big((u_0 - \varepsilon)\psi_1 + u_{\overline{1}}\psi_0\Big) + \gamma'^- \psi_1$$

Takagi-Taupin description in elliptical geometry:
incoming wave and reflected wave
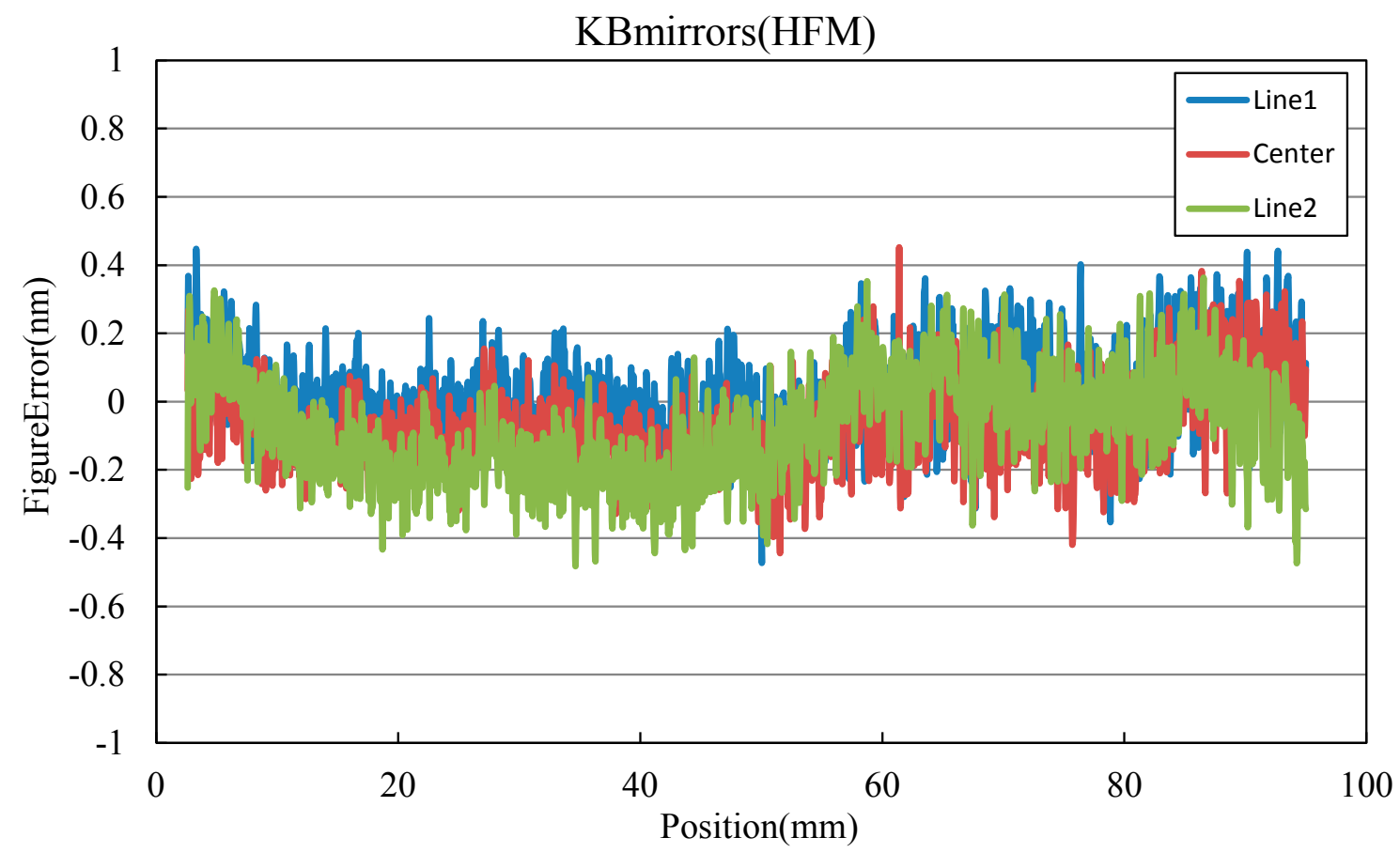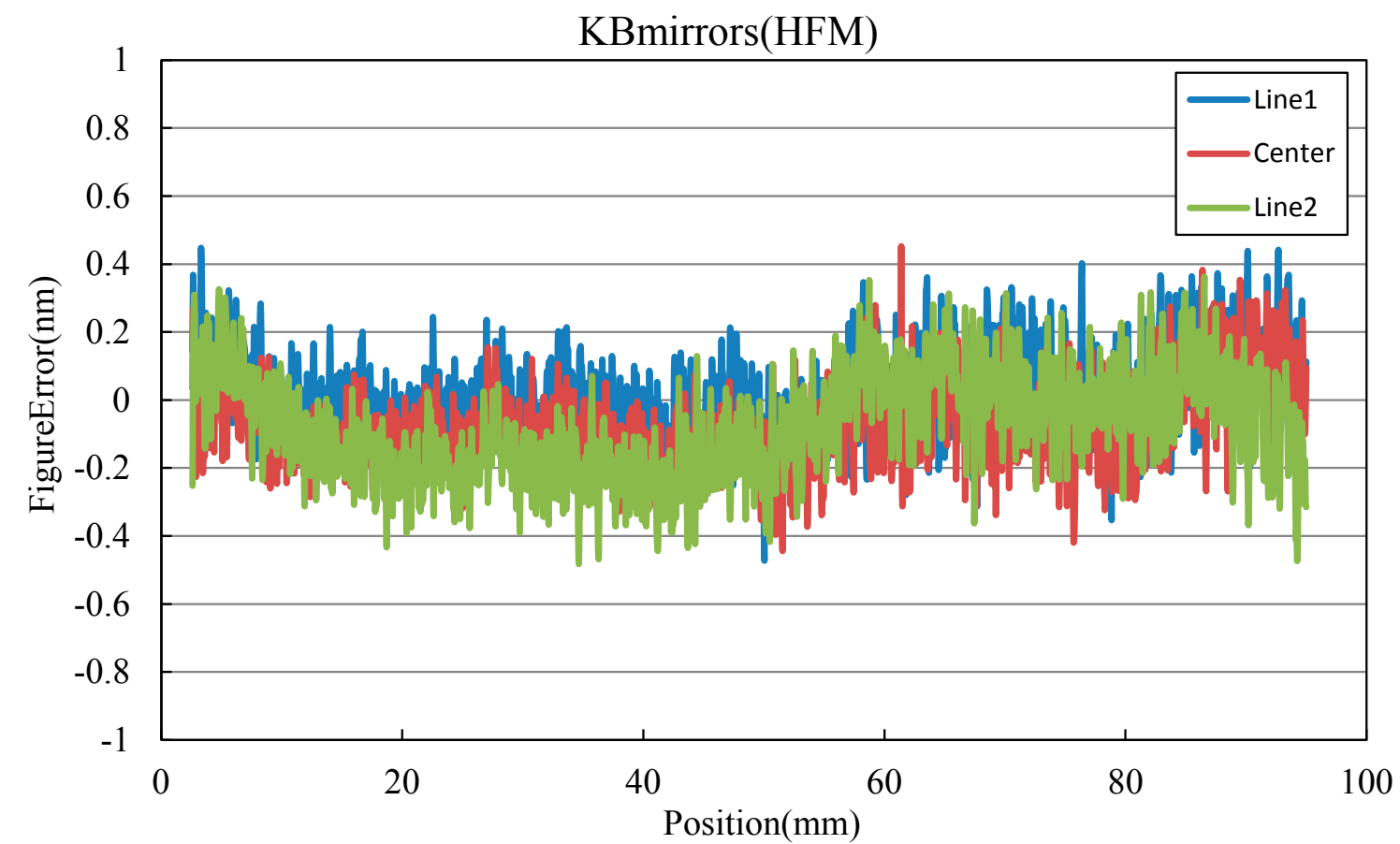are coupled by quasi-periodic susceptibility

Osterhoff, Morawe, Ferrero, Guigay: Optics Letters, 2012

Recent Progress

Figure 2-3: Tangential Figure error profiles of HFM

▶ smooth surfaces
   sub-nm figure error

JTEC, Osaka

Recent Progress

Figure 2-3: Tangential Figure error profiles of HFM

► smooth surfaces
  sub-nm figure error

► simulations to
  single nano metre

JTEC, Osaka
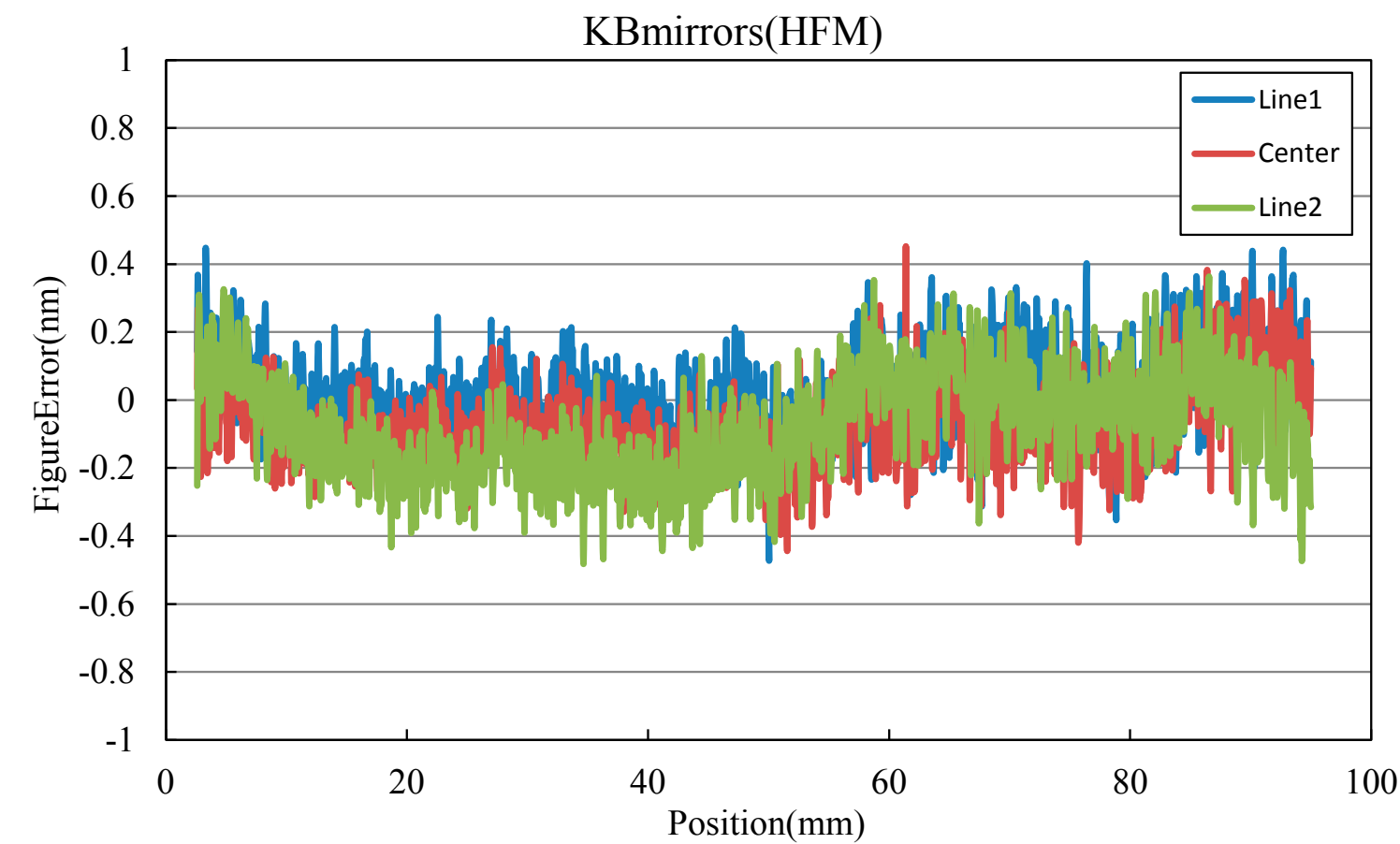Osterhoff, PhD thesis, 2011

Recent Progress

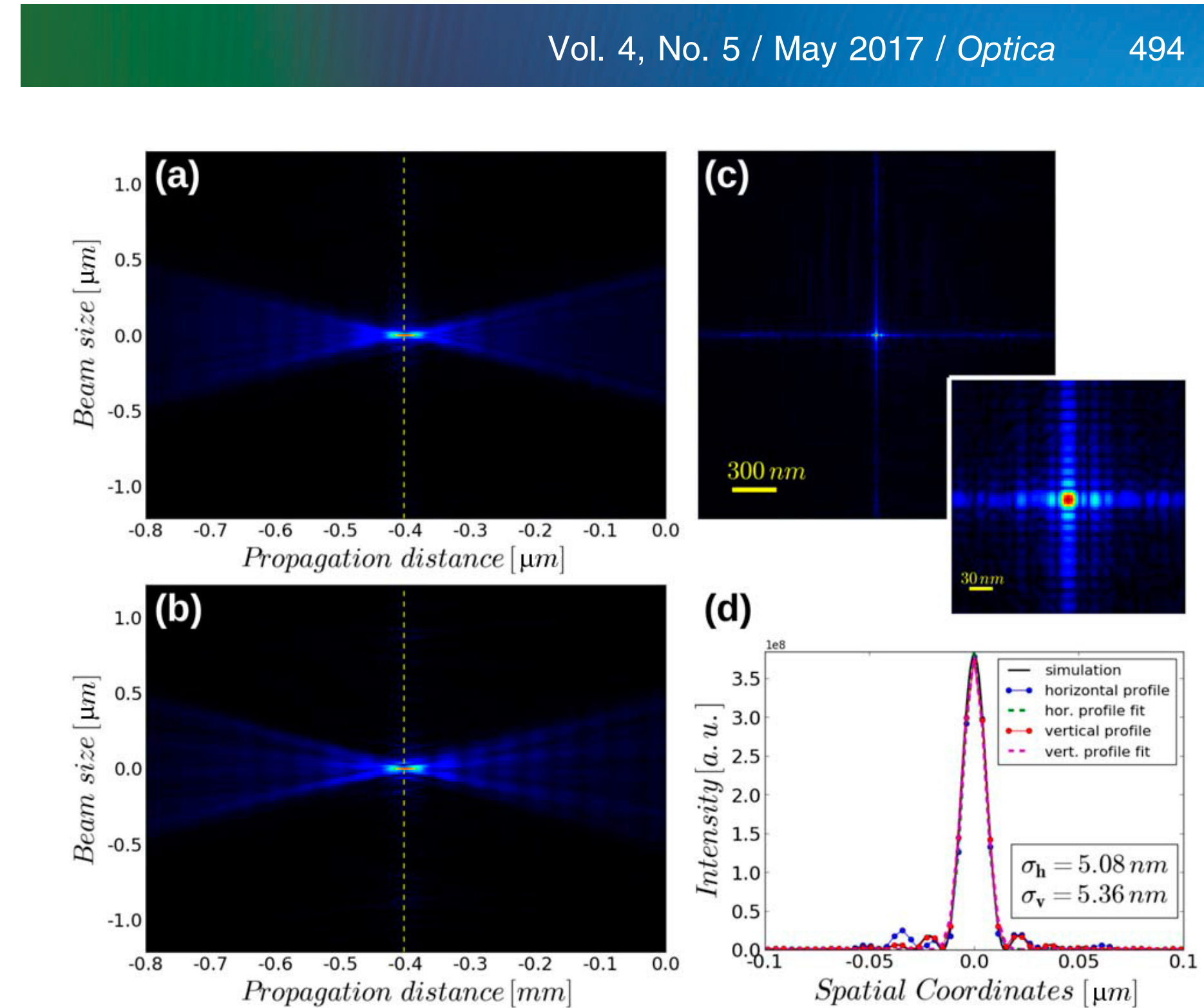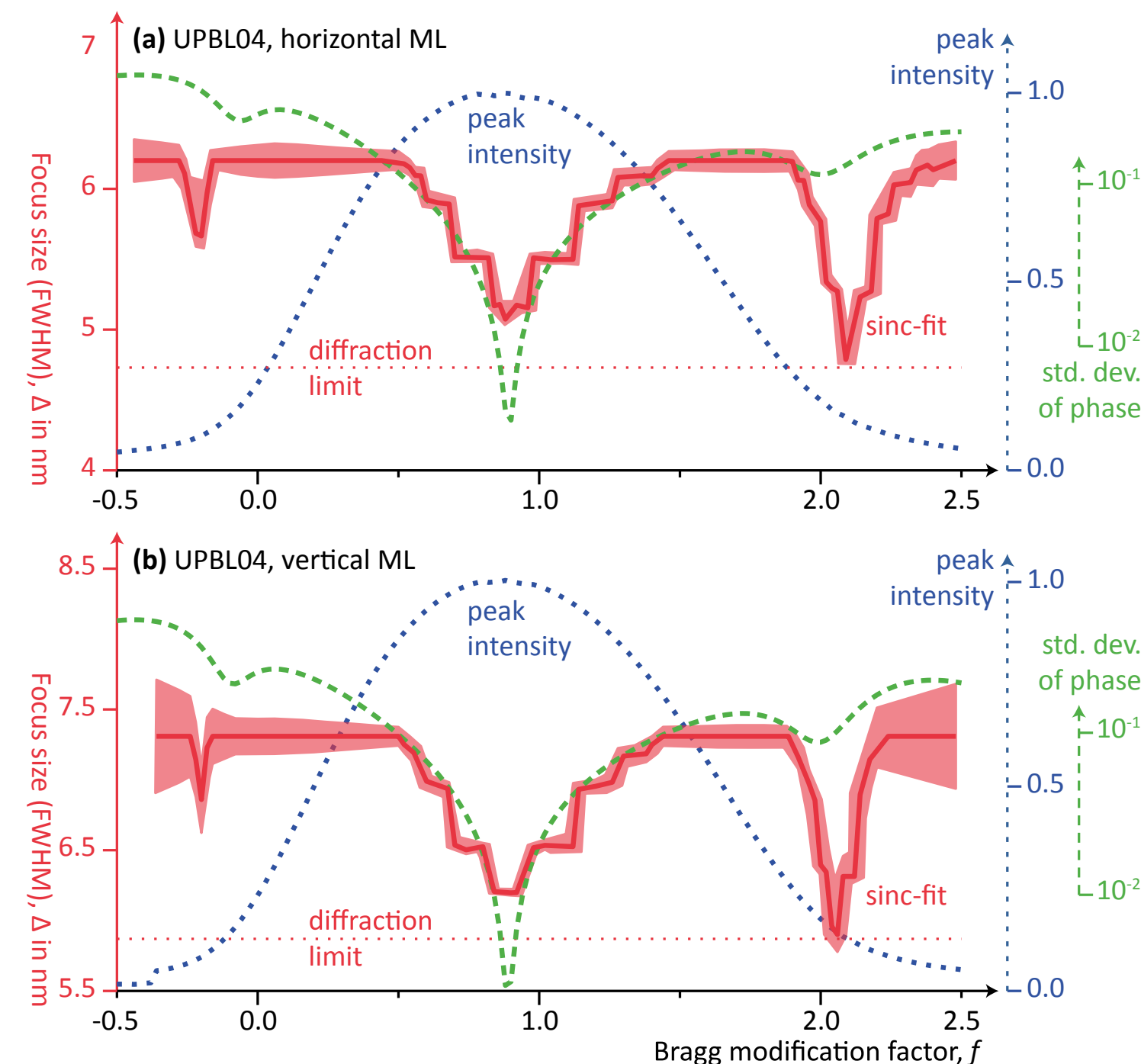Figure 2-3: Tangential Figure error profiles of HFM

Fig. 3. X-ray wavefront characterization and focus size. (a) Vertical cross-section of the propagation of the wavefront. (b) Horizontal cross-section of the propagation of the wavefront. In (a) and (b), the sample is at position 0 and the focus is indicated by the dashed yellow line. (c) Beam at the focus position. The inset displays a zoomed-in view of the central part. (d) Horizontal and vertical profiles of the intensity of the beam at the focus position plotted together with a simulated profile of an ideal optical system. The FWHM values calculated by $2\sqrt{2\log(2)}\sigma$ were 12.0 nm for the horizontal focus and 12.6 nm for the vertical focus.

► smooth surfaces
  sub-nm figure error

► simulations to
  single nano metre

► ML mirrors down to
  12 nm, above 20 keV

12.0 nm ×
12.6 nm
(FWHM),
≥ 20 keV

JTEC, Osaka
Osterhoff, PhD thesis, 2011
da Silva, Cloetens et al, Optica, 2017

Fresnel Zone Plates, ML Laue Lenses, ML Zone Plates

A set of concentric ellipses along with a depiction of a multilayer Laue lens.

A schematic wedged multilayer Laue lens in which each interface is slanted to achieve a Bragg condition.

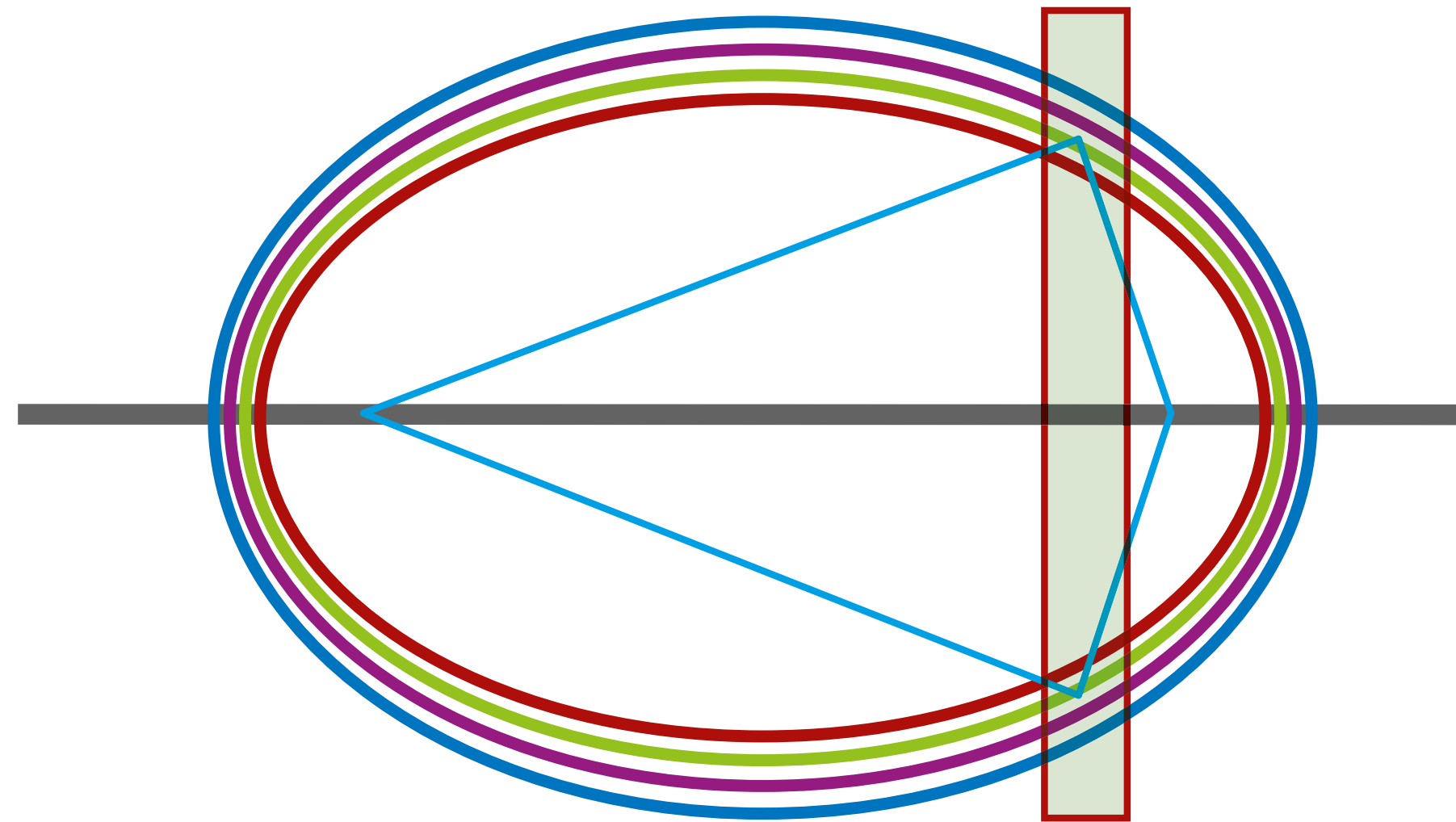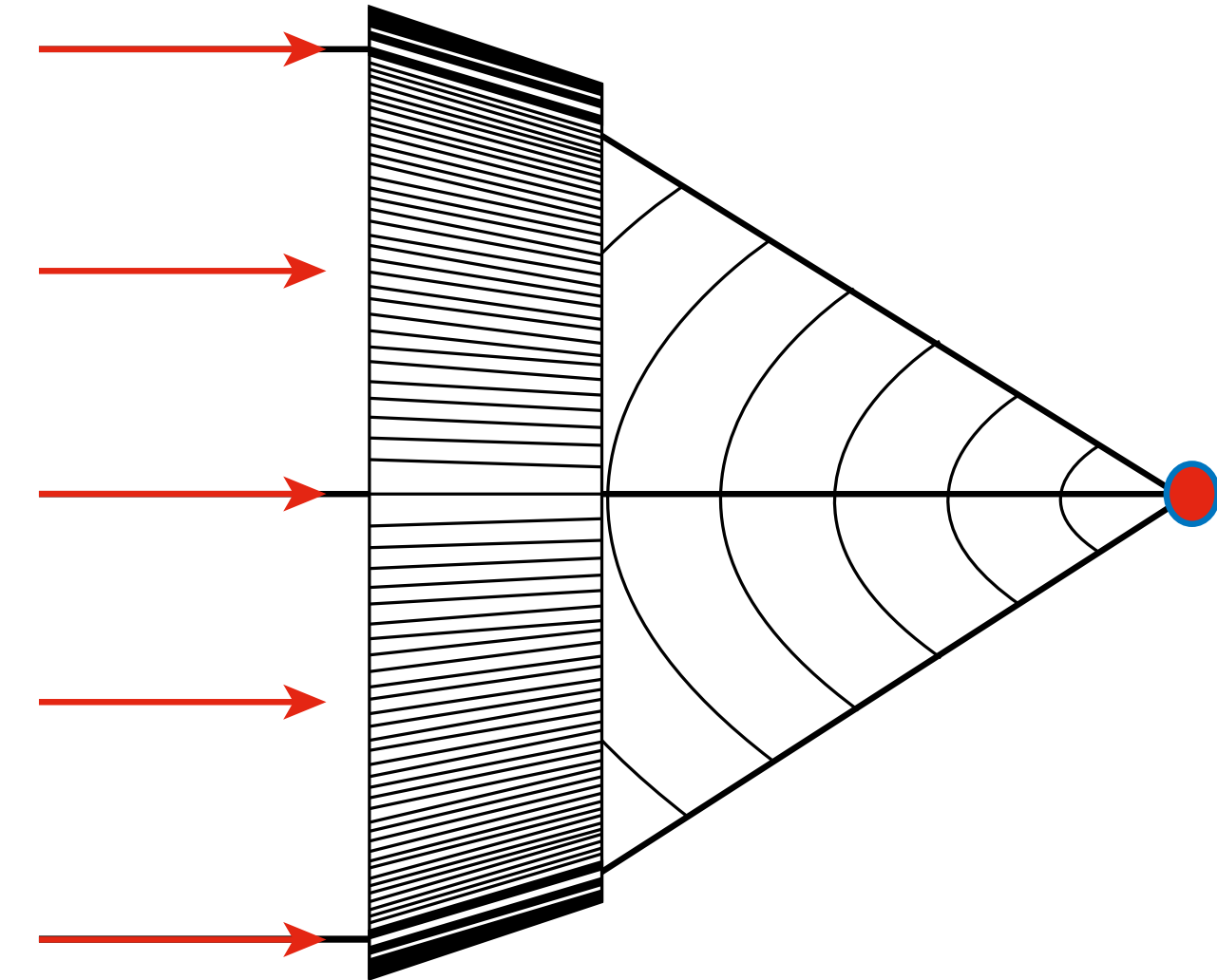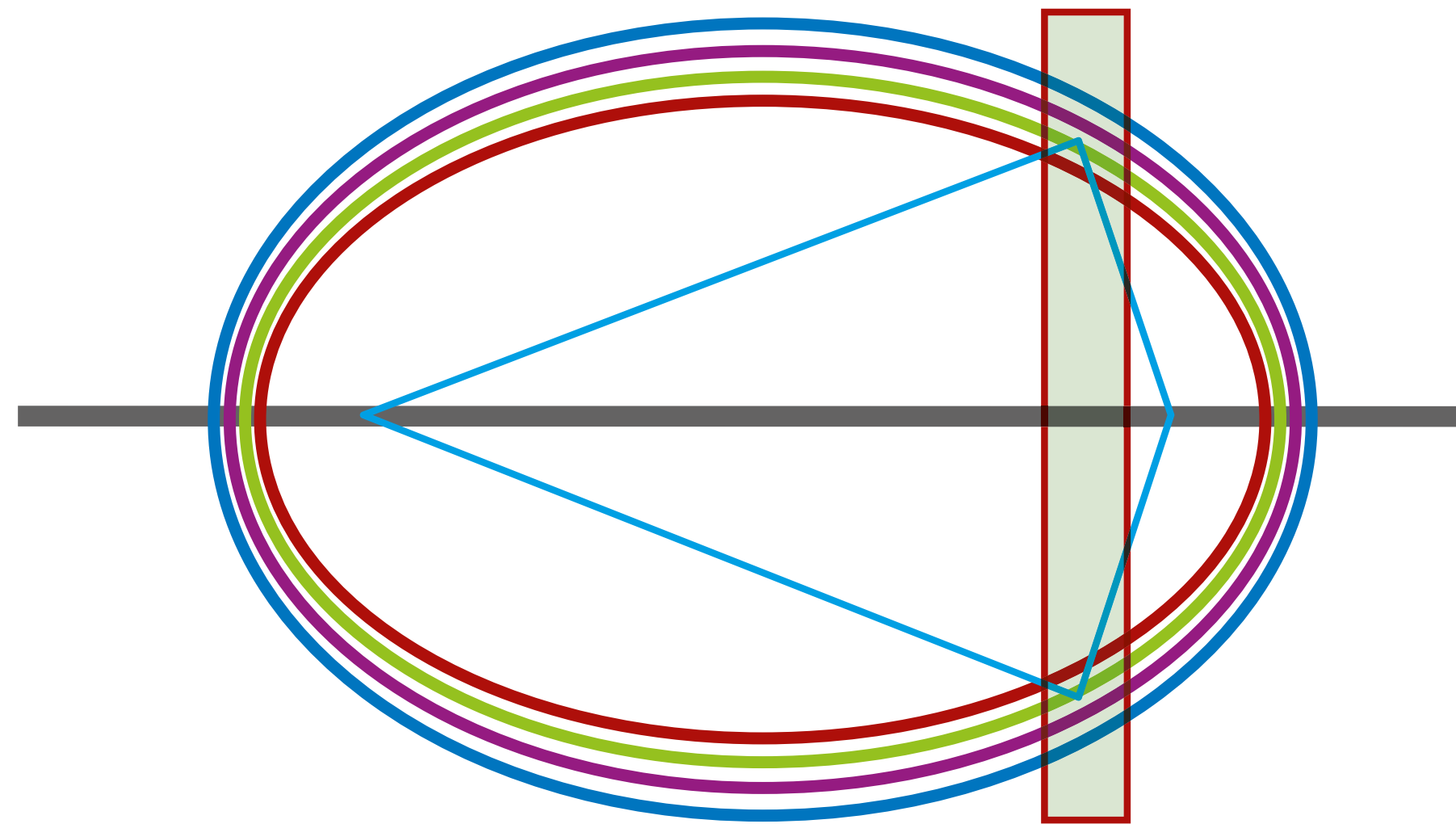$$r_n = \sqrt{(n\lambda/2)^2 + n\lambda f}$$

Figure 2

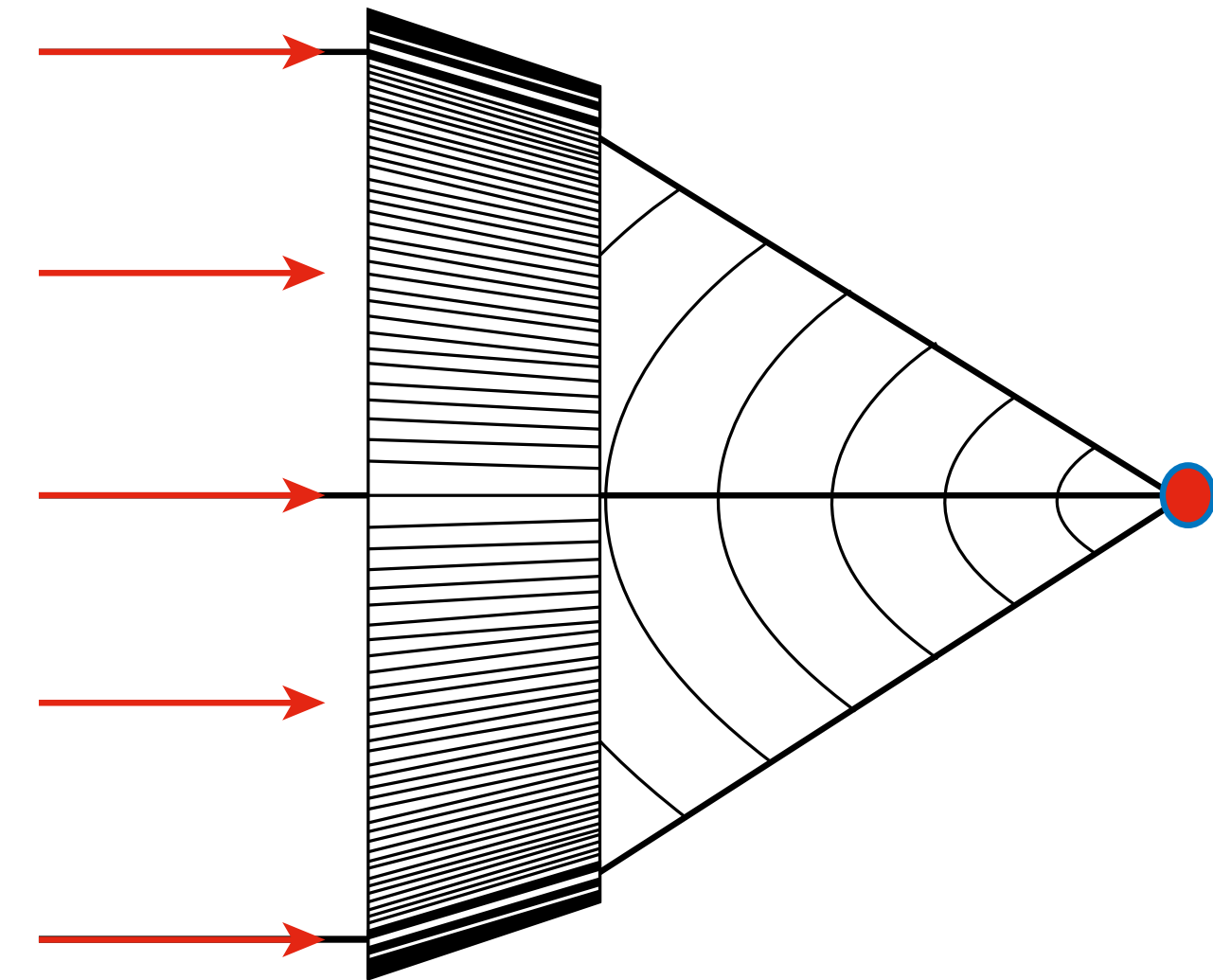A set of concentric ellipses along with a depiction of a multilayer Laue lens.

Figure 10

A schematic wedged multilayer Laue lens in which each interface is slanted to achieve a Bragg condition.

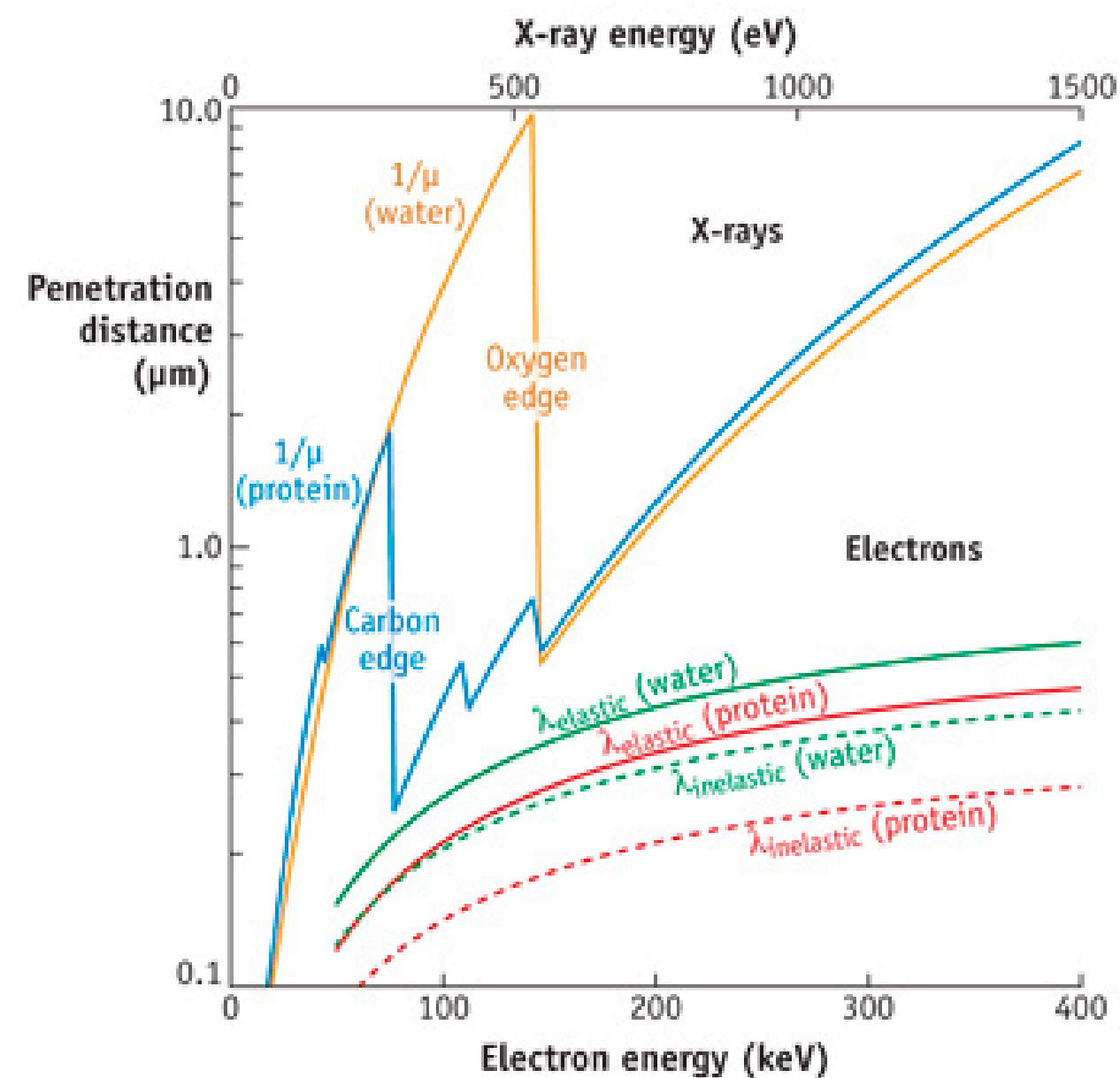$$r_n = \sqrt{(n\lambda/2)^2 + n\lambda f}$$

Zone Plate Laws

▶ resolution given by outermost zone width

▶ zone width limited by fabrication

▶ efficiency limited by optical thickness

▶ thin and long zones: volume diffraction; "Bergemann-Limit"

Macrander, Huang: Annual Review of Material Research, '17

Water Window

► high contrast
between
C (cell) and
O (water)

► soft x-rays,
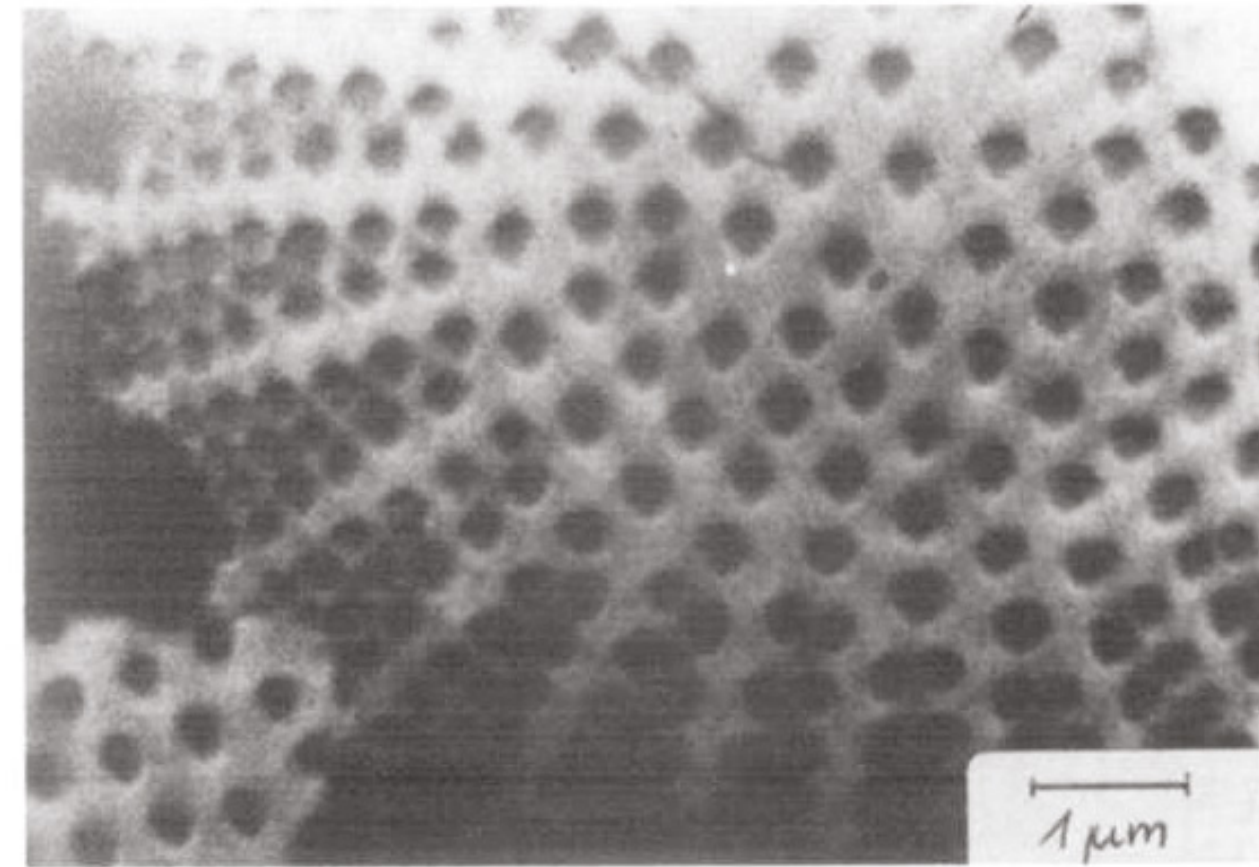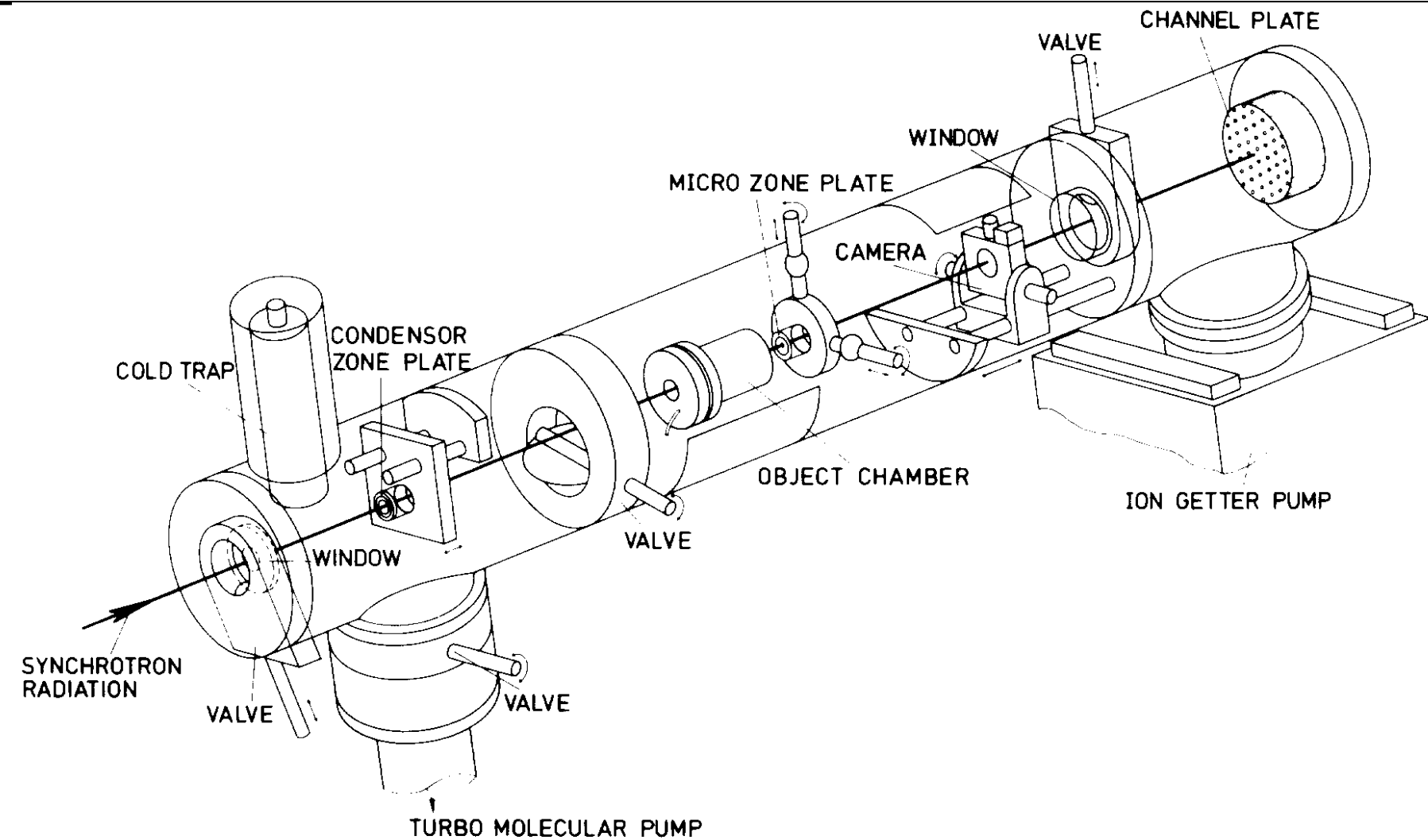good optics

http://www.atto.ethz.ch/research/timeresolved.html

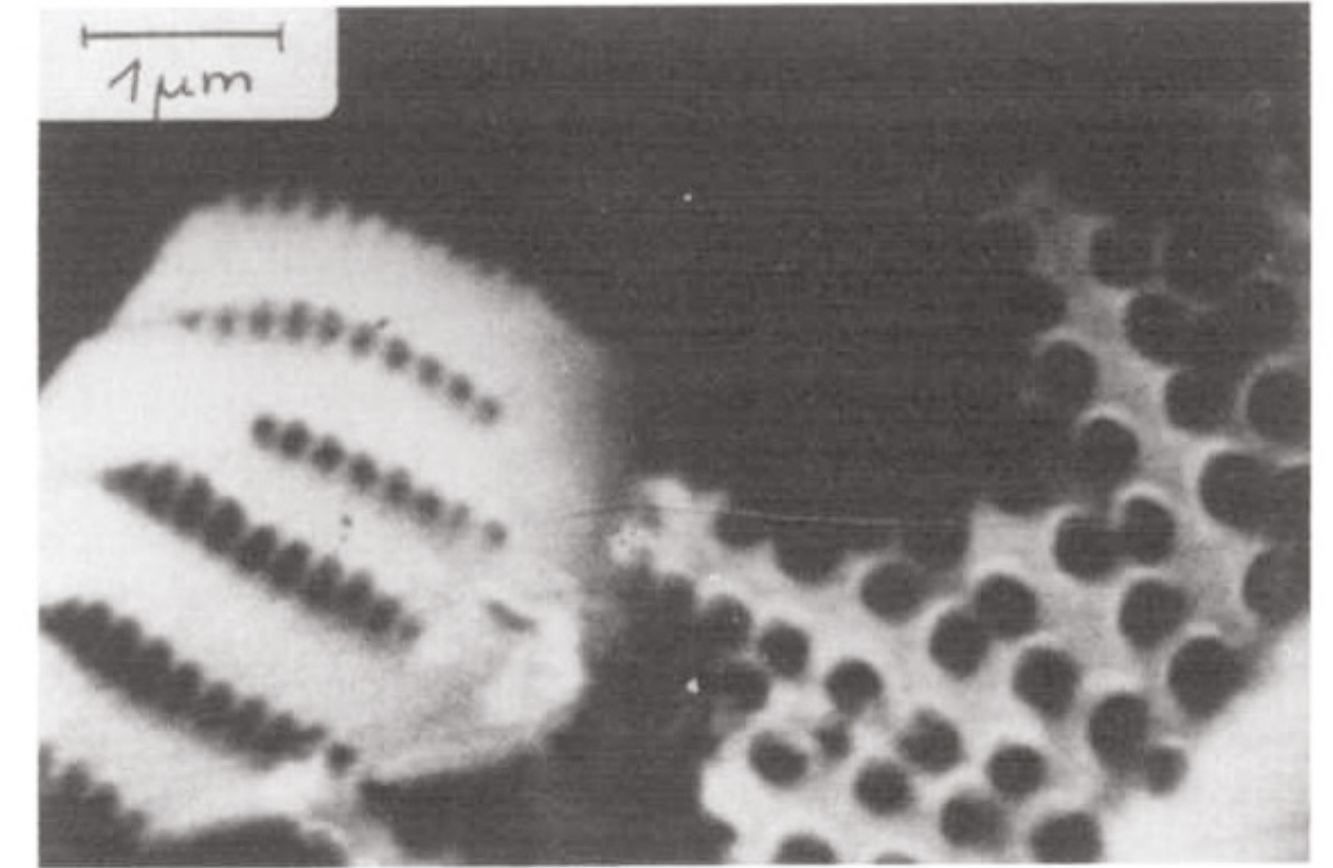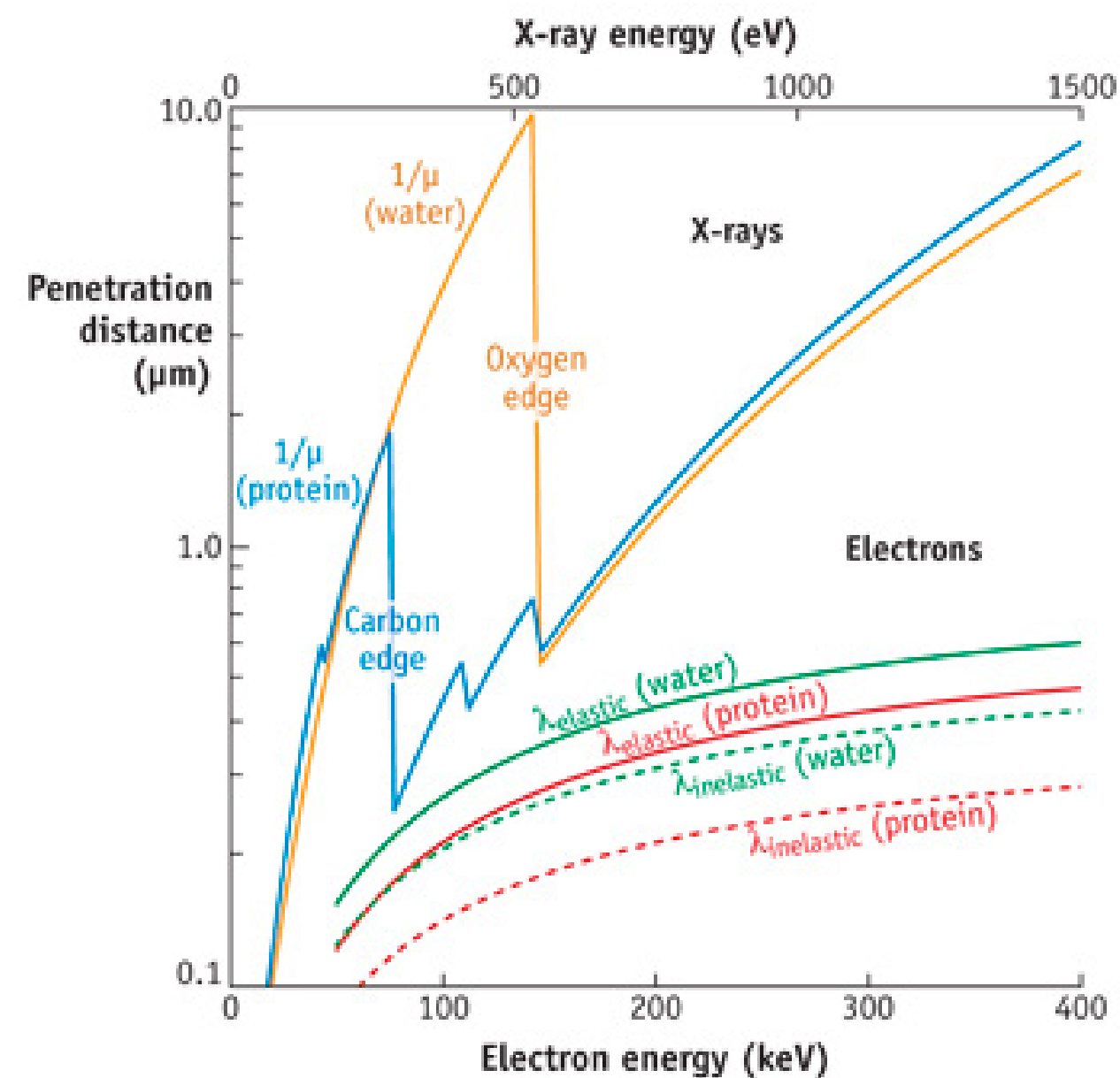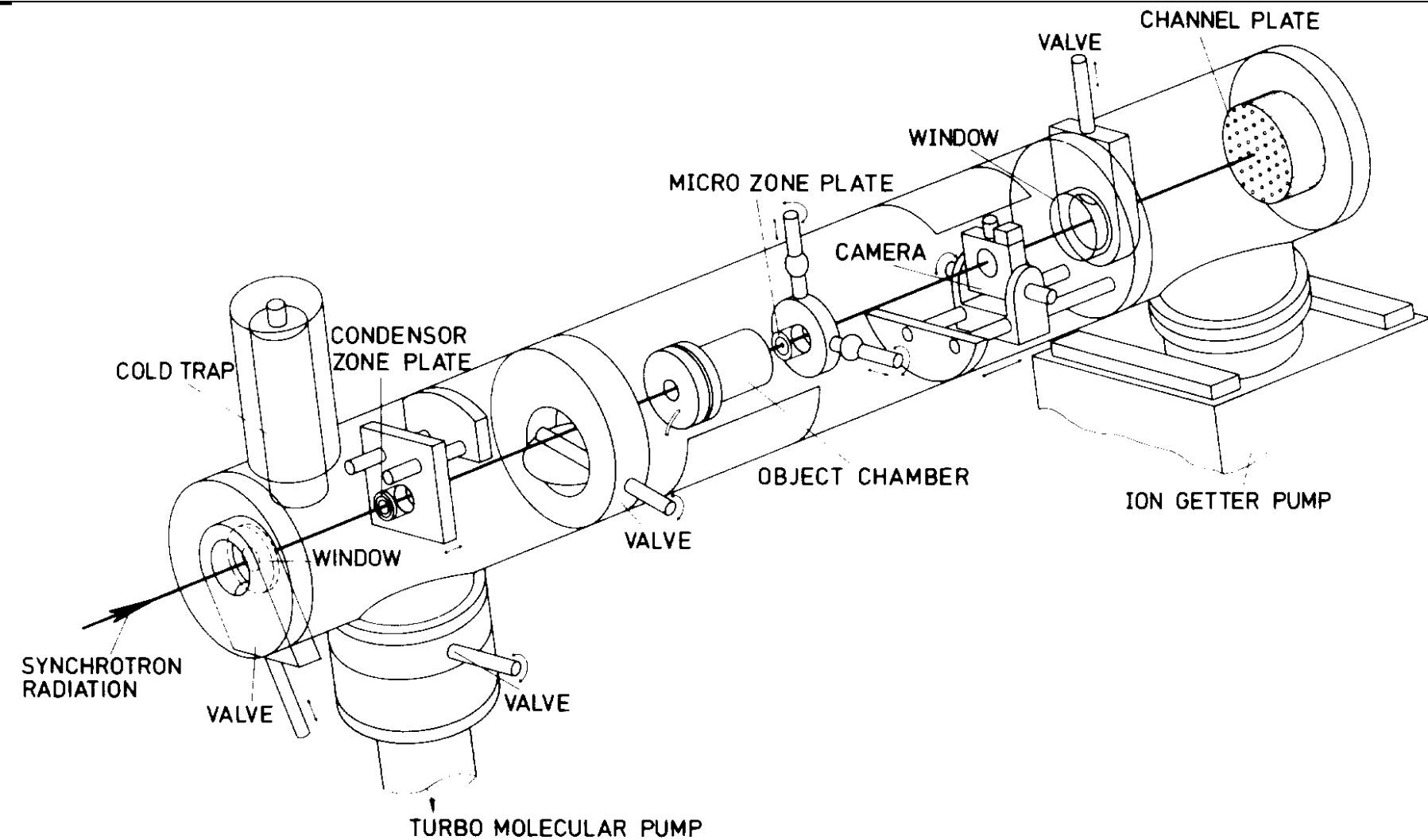Fig. 4. Diatom, imaged with 4.5 nm X-rays, X-ray magnification 250×, exposure time 20 s, storage ring current 100 mA.

Fig. 6. Diatoms, imaged with 4.5 nm X-rays, X-ray magnification 500×, exposure time 60 s, storage ring current 150 mA.

Water Window

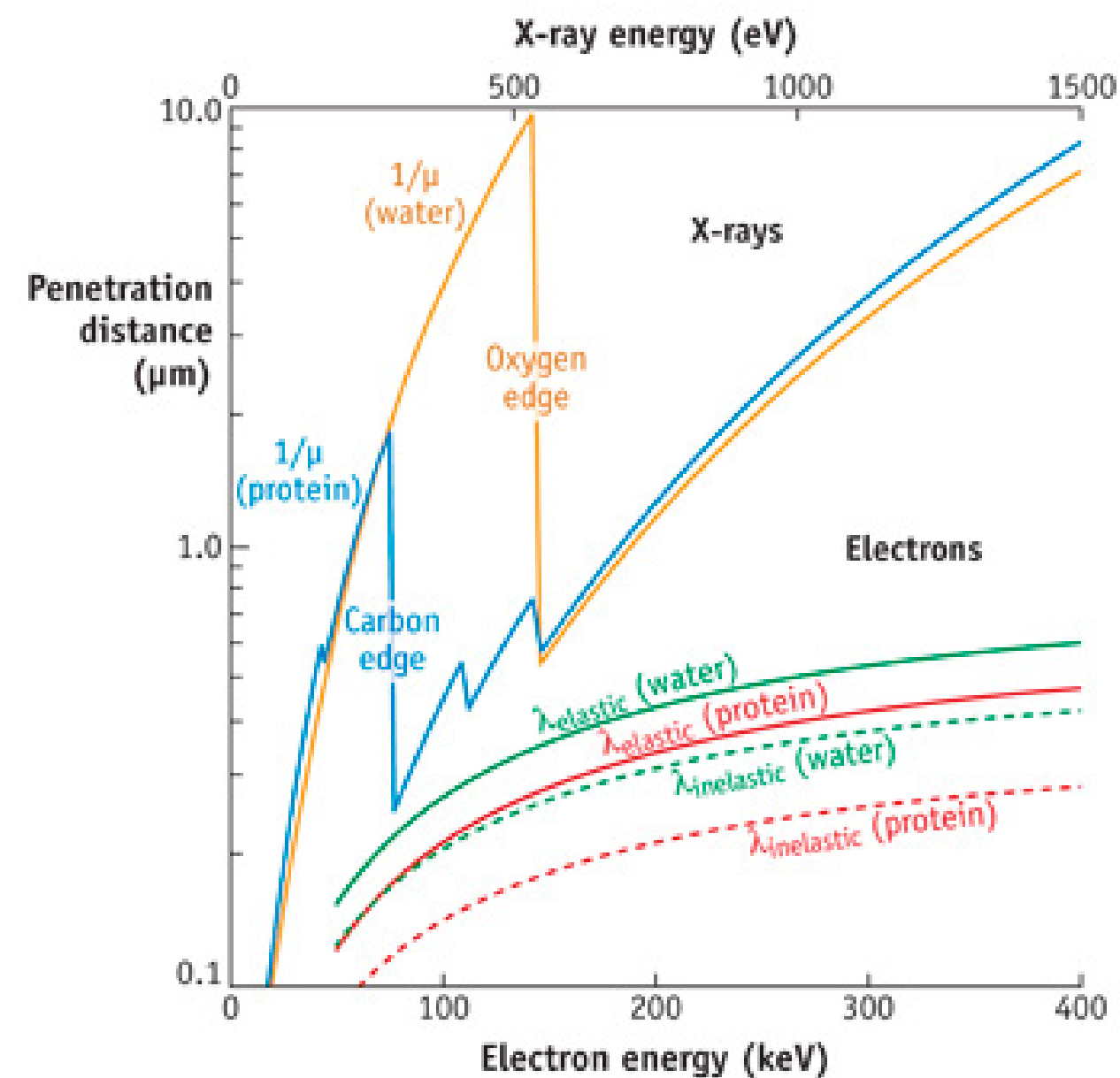▶ high contrast between C (cell) and O (water)
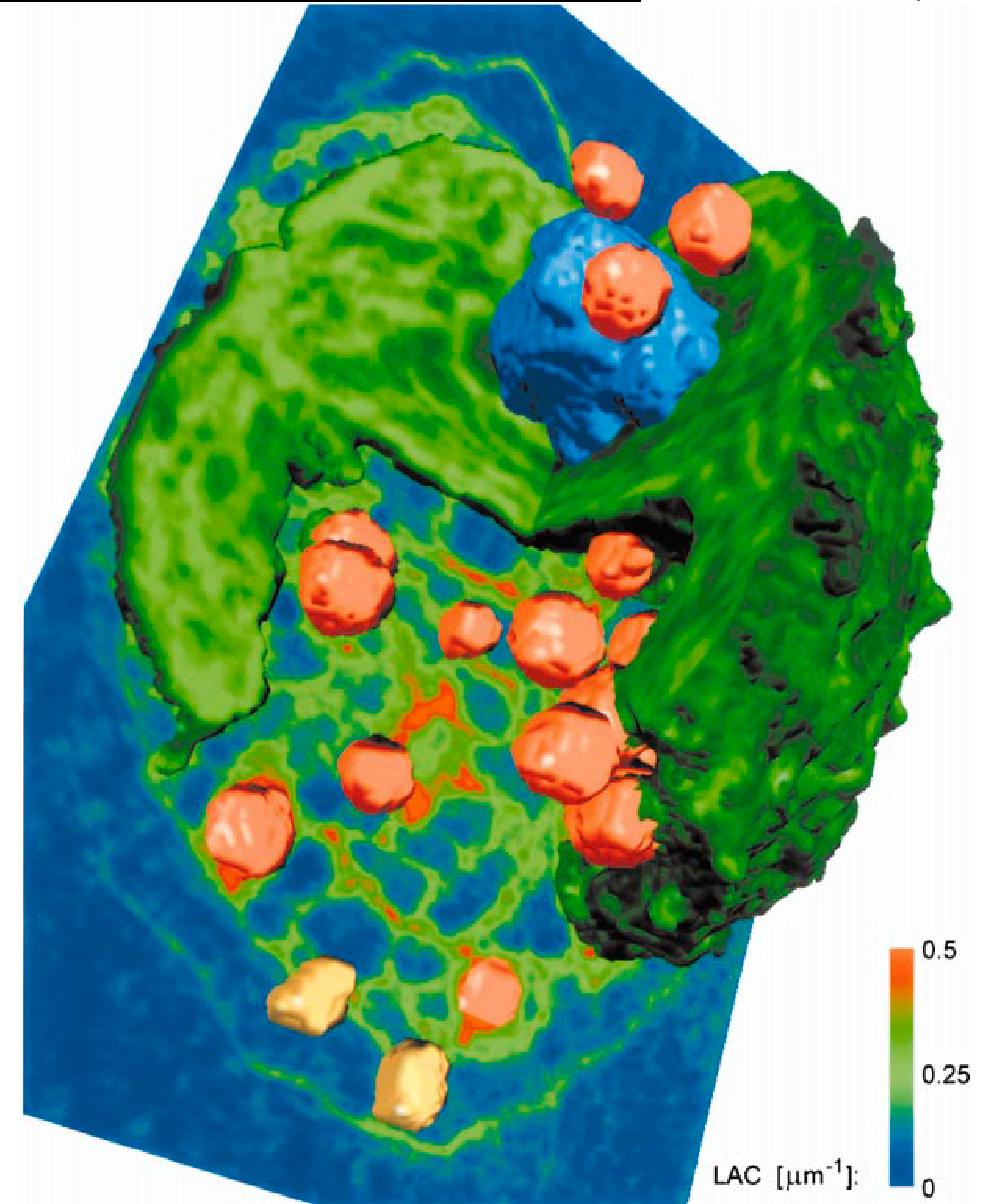
▶ soft x-rays, good optics

Water Window

► high contrast between C (cell) and O (water)

► soft x-rays, good optics

http://www.atto.ethz.ch/research/timeresolved.html

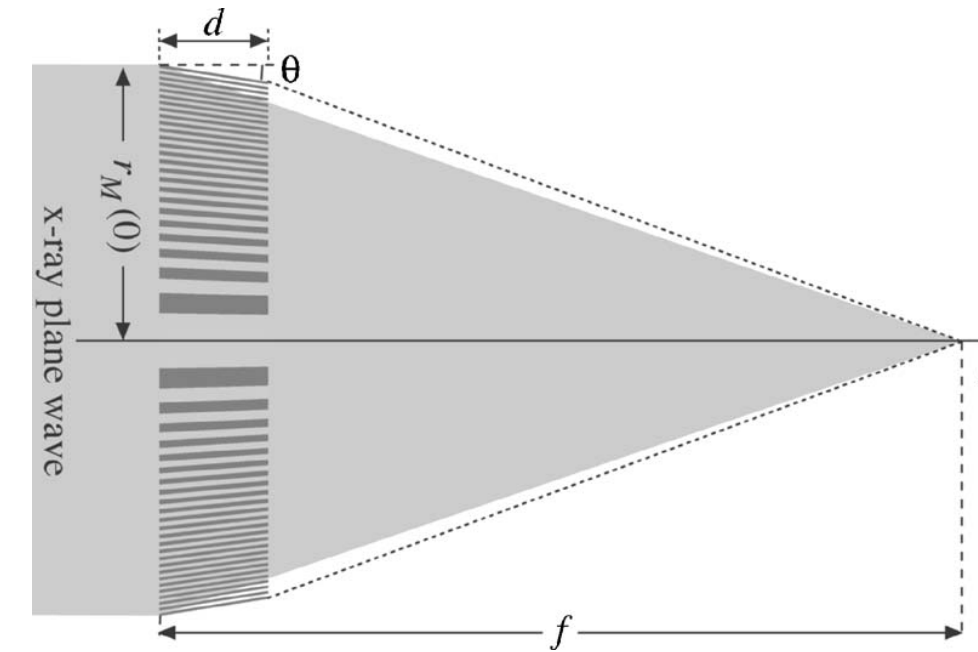Weiß, Schmahl et al: Ultramicroscopy, 2000

FIG. 1. Focusing geometry for a thick Fresnel zone plate with tilted zones.

FIG. 2. First-order focus of a parabolically tilted zone plate (solid line), an equivalent ideal thin zone plate (dotted line), and a straight (not tilted) zone plate (dash-dotted line). The inset shows the beam intensity distribution around the focus along the optical axis for the parabolically tilted zone plate.

Efficiency for hard x-rays?

► vanishing absorption
no absorption-based zone plate

► small phase shift per μm
optically thick, "long" zone plates needed

► aspect ratio ~ 1:1000 and more
outermost zone width vs. optical thickness

Schroer, Phys Rev B, 2006

(a) *MZP fabrication*

pulsed laser deposition

multilayer zone plate

dr

MZP focus

b

focused ion beam

t $f = 50\ \mu m$

z

(b) *experimental setup*

KB mirror pair

defocus
2 mm

beamstop

KB focus

detector

Fig. 1. (a) Schematic fabrication process: Pulsed laser deposition of W and Si multilayer onto a rotating wire according to the Fresnel zone plate law. Focused ion beam fabrication of the MZP by cutting a slice out of the coated wire, placing it onto a sample holder and polishing it down to the optimal optical thickness of 0.7 μm. (b) Experimental setup of the synchrotron experiment: The MZP is positioned 2 mm downstream of the KB focus.

10 nm

FIG. 1. Focusing geometry for a thick Fresnel zone plate with tilted zones.

d θ

$r_M(0)$

x-ray plane wave

z
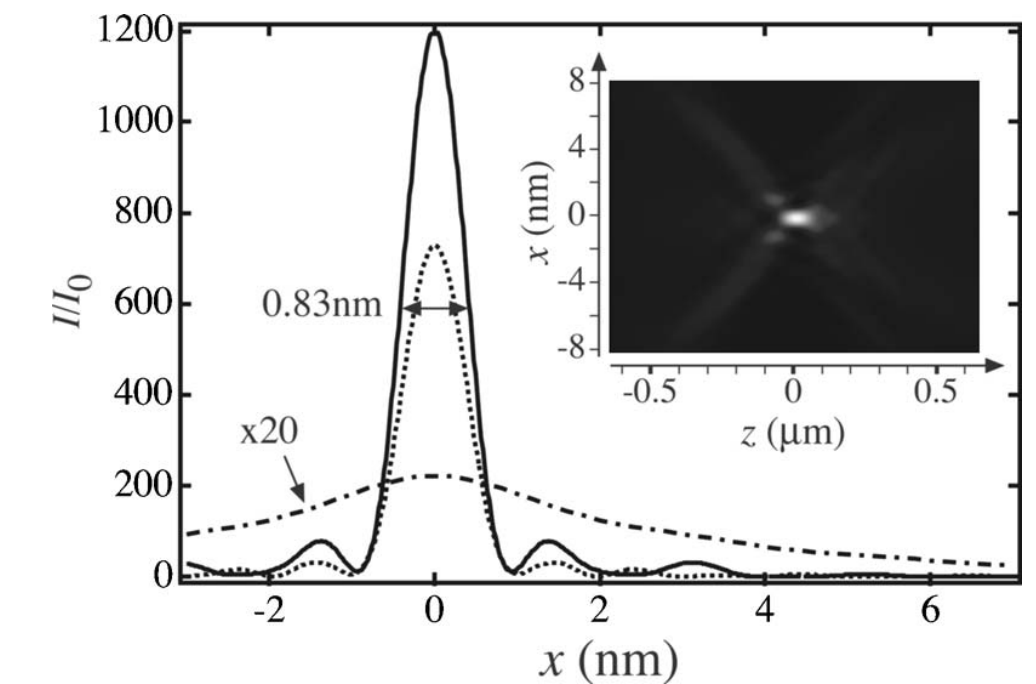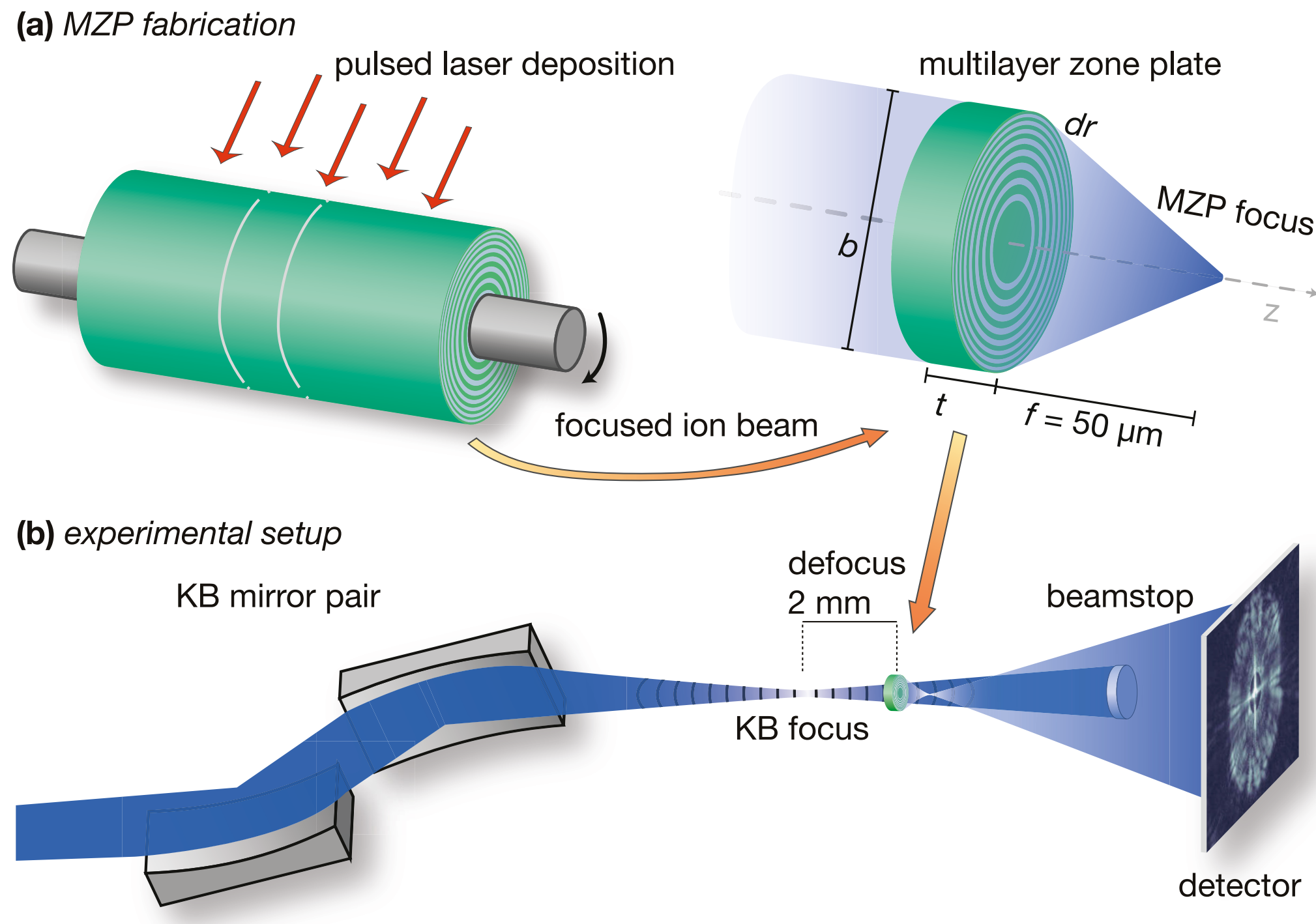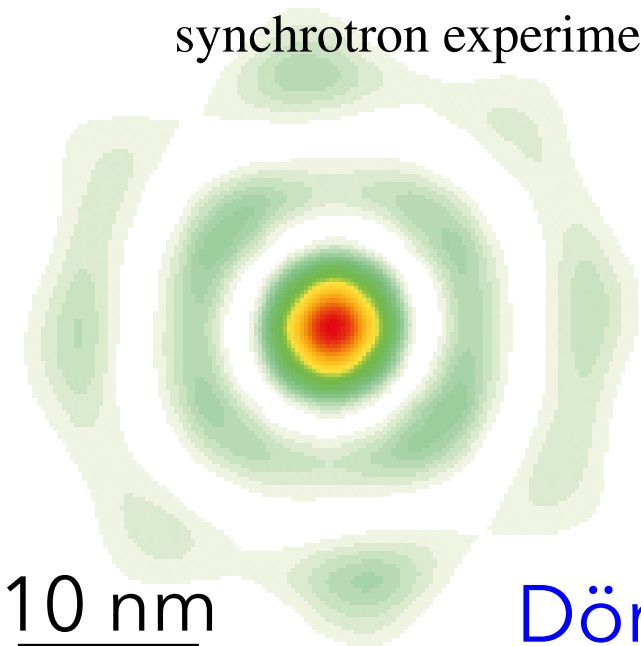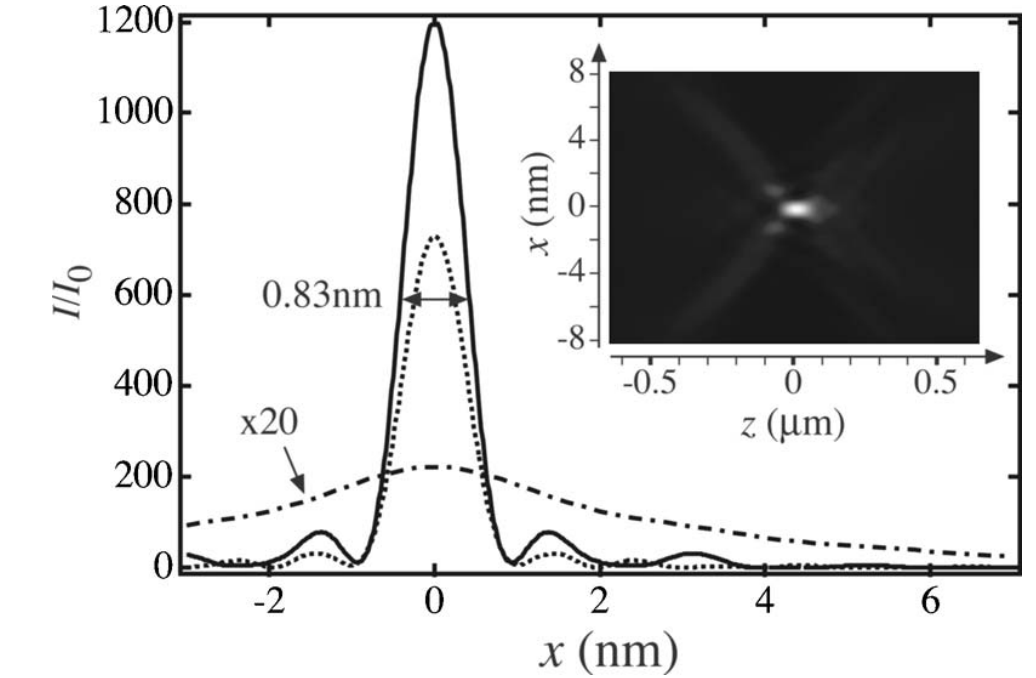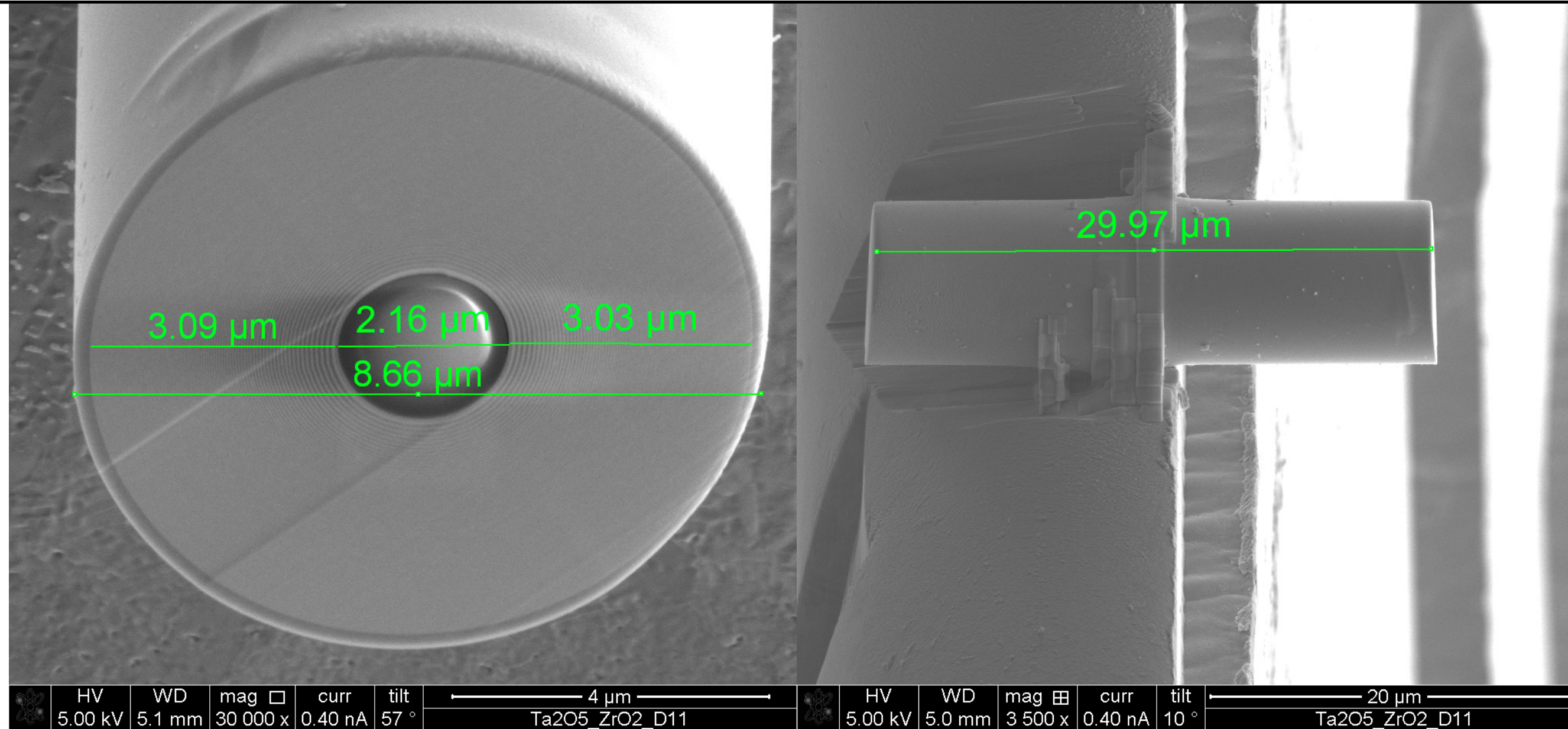
f

I/I_0

0.83nm

x20

x (nm)

x (nm)

z (μm)

FIG. 2. First-order focus of a parabolically tilted zone plate (solid line), an equivalent ideal thin zone plate (dotted line), and a straight (not tilted) zone plate (dash-dotted line). The inset shows the beam intensity distribution around the focus along the optical axis for the parabolically tilted zone plate.

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Döring, Osterhoff et al, Optics Express, 2013   Schroer, Phys Rev B, 2006
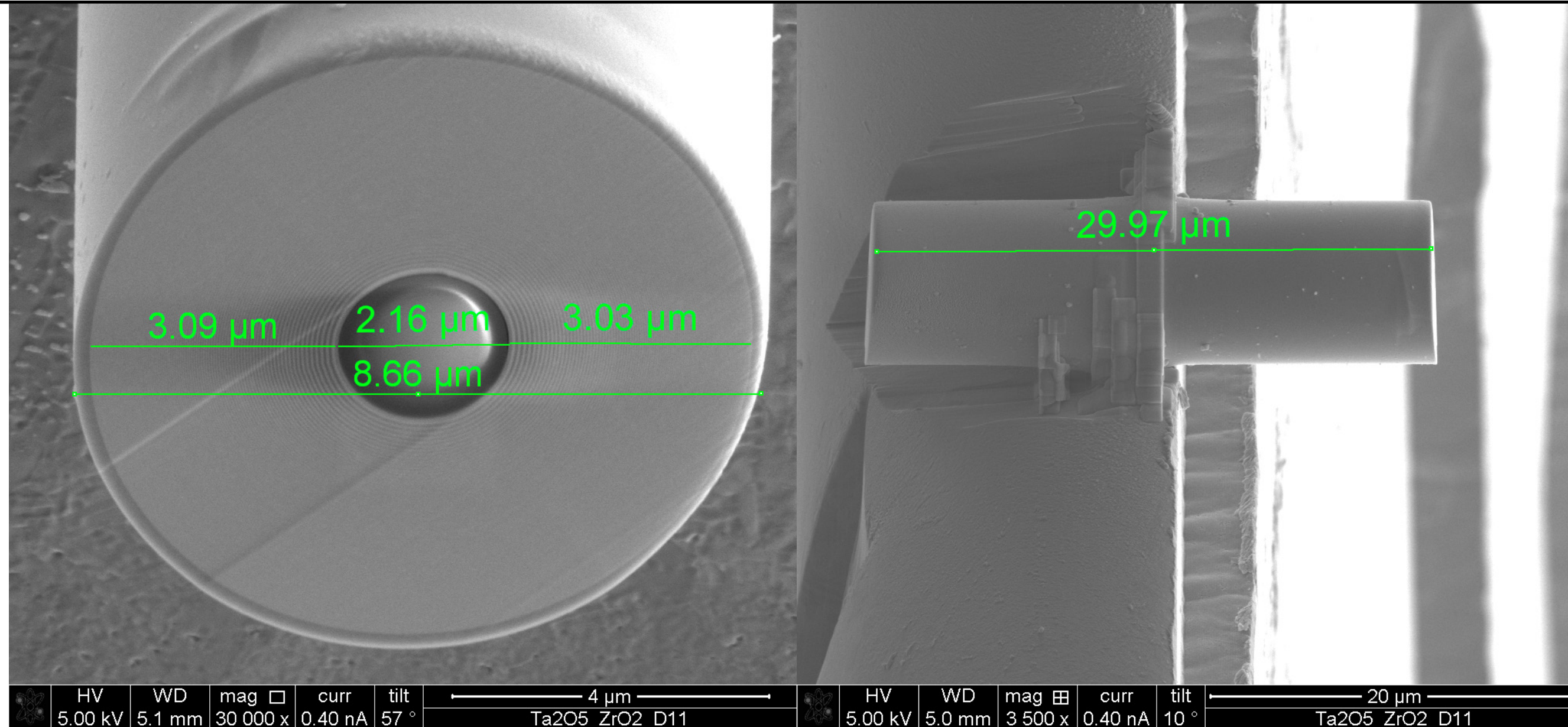
► Multilayer Zone Plate for high x-ray energies designed for 60 keV, tested also at 100 keV

► outermost zones: 10 nm

► optical thickness: 30 µm

Eberl, Soltau, Osterhoff et al (work in progress)

Figure 1. (**a**) A wedged multilayer Laue lens of focal length *f* is constructed from layers whose spacing follows the zone-plate condition. To achieve high efficiency the lens must be thick, in which case diffraction is a volume effect described by dynamical diffraction. In this case the layers should be tilted to locally obey Bragg's law, which places them normal to a circle of radius 2*f*. (**b**) SEM image of the 2750-bilayer wedged MLL used in this study. The regions corresponding to the multilayered materials and the Si substrate are indicated. The white scale bar is 20 μm and the inset shows a magnified TEM image of the layered materials.

► Multilayer Zone Plate for high x-ray energies designed for 60 keV, tested also at 100 keV

► outermost zones: 10 nm

► optical thickness: 30 μm

Eberl, Soltau, Osterhoff et al (work in progress)

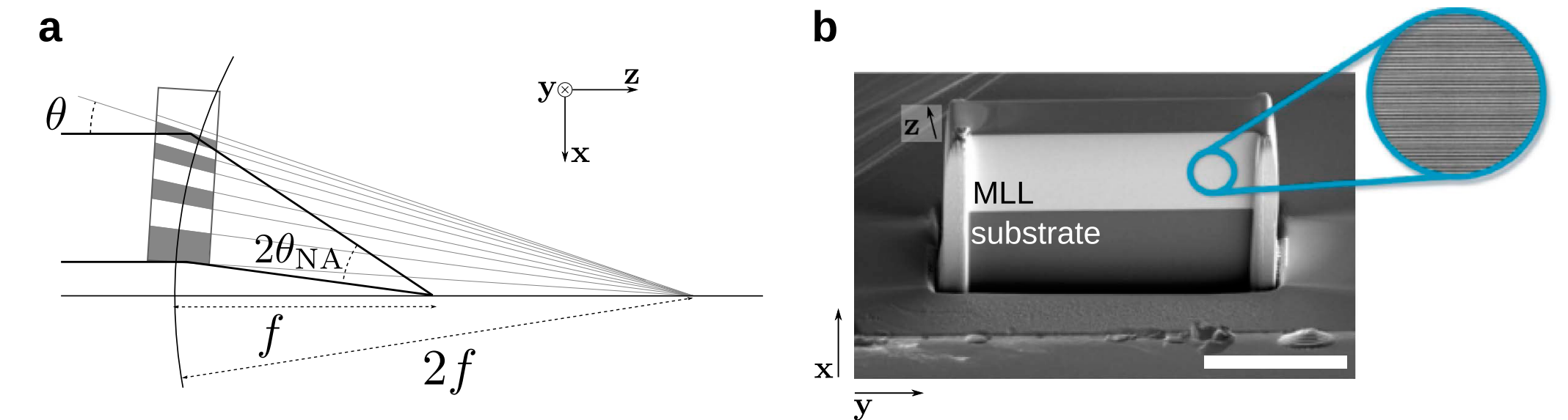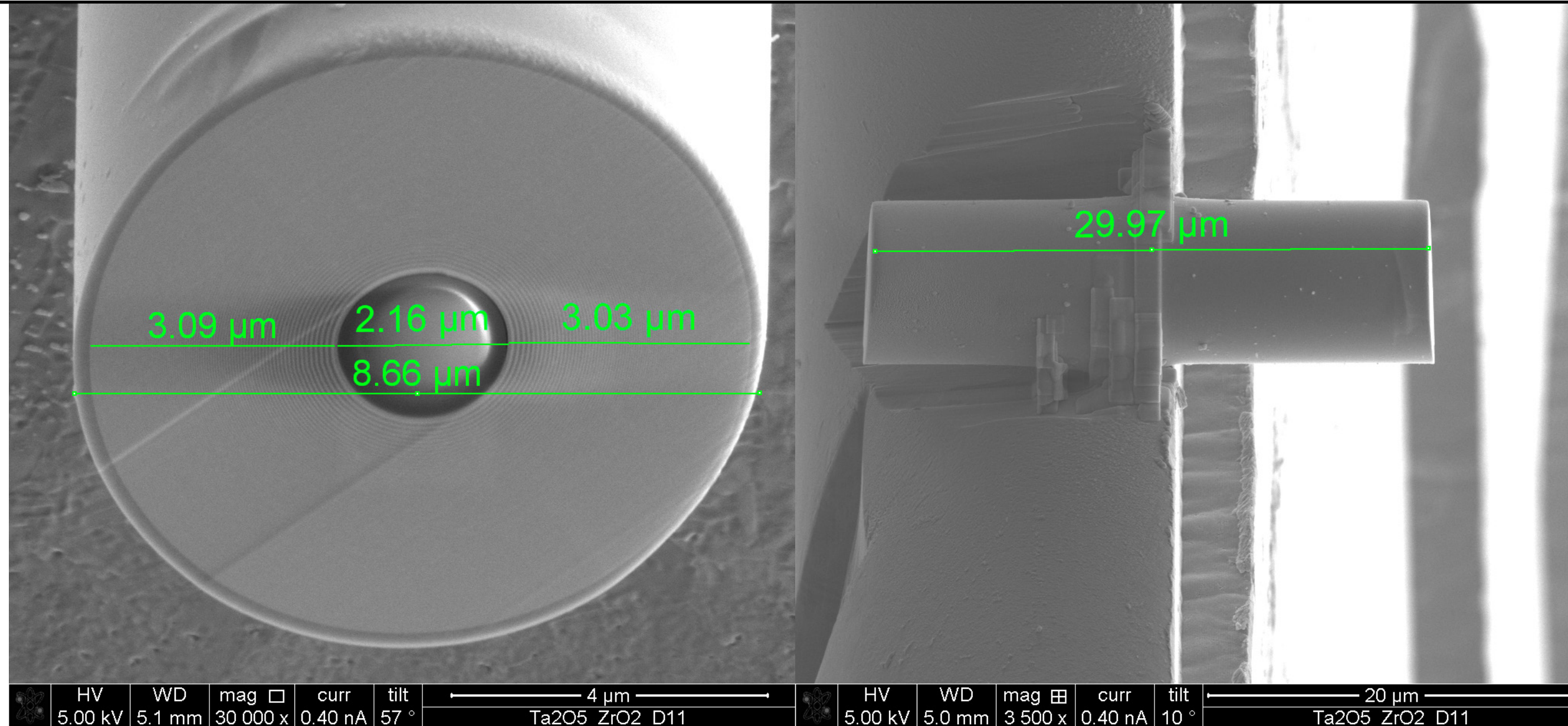Bajt et al, Scientific Reports, 2015

► Multilayer Zone Plate for high x-ray energies designed for 60 keV, tested also at 100 keV

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Eberl, Soltau, Osterhoff et al (work in progress)

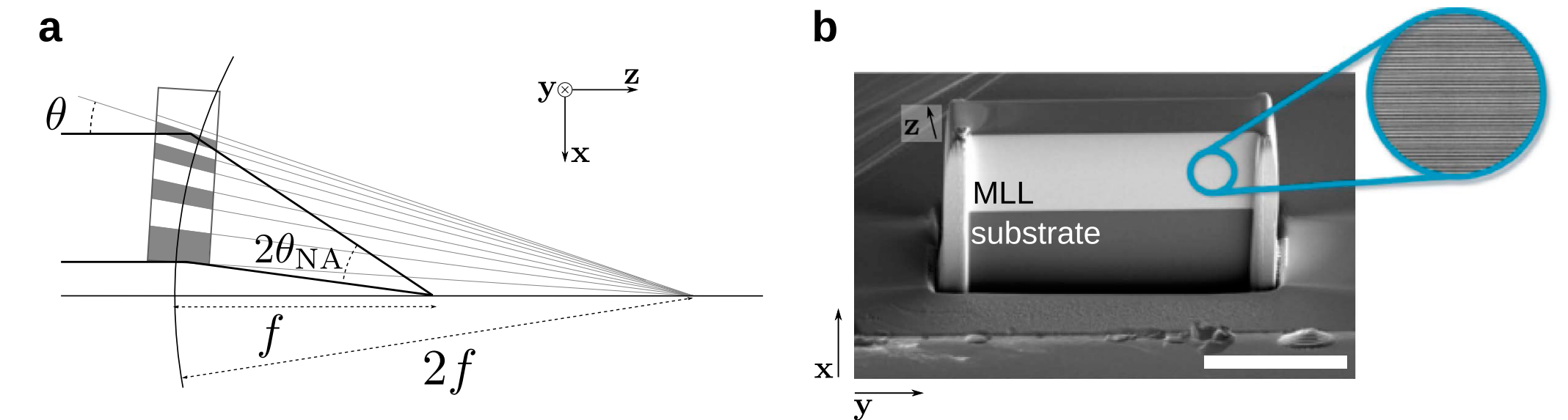Bajt et al, Scientific Reports, 2015
Gleber et al, Optics Express, 2014

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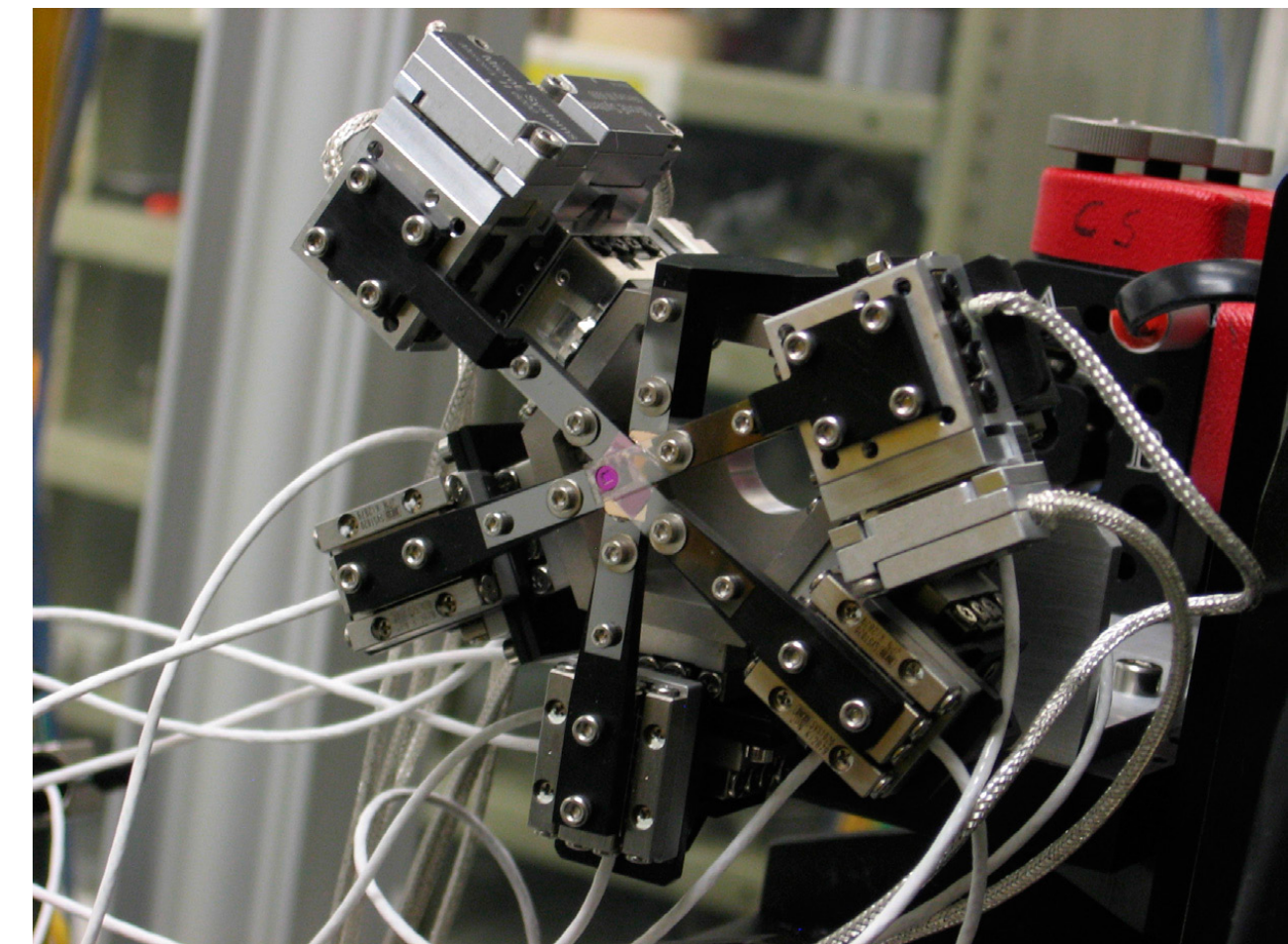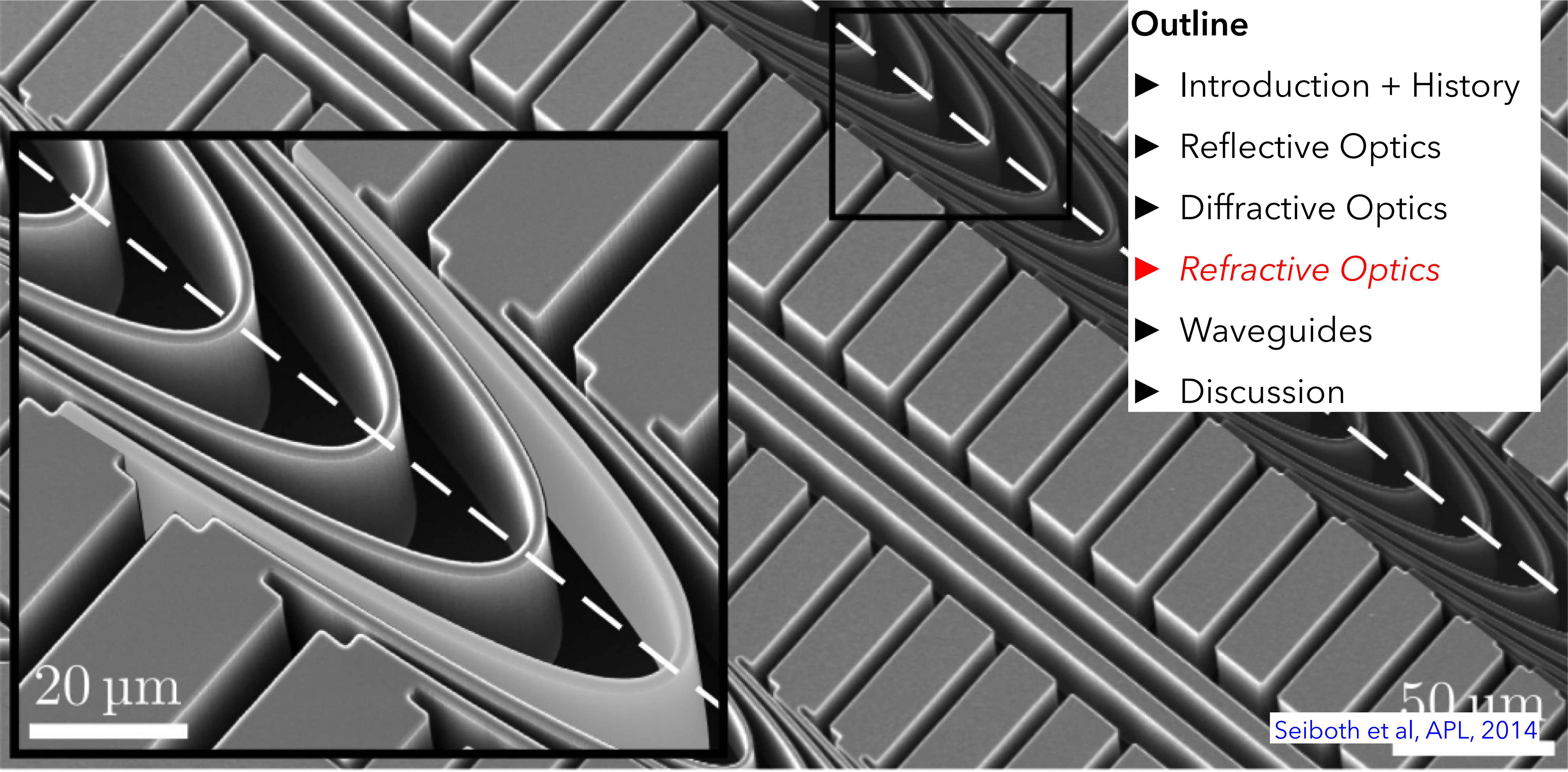

Fig. 4. Picture of the ANL Z2-37 precision alignment apparatus for intermediate-field stacking of up to six zone plates. In-house fabricated arrays of Fresnel zone plates are mounted on diamond holders. The apparatus is shown as integrated to the microprobe setup. [D. Shu, J. Liu, S. C. Gleber, J. Vila-Comamala, B. Lai, J. Maser, C. Roehrig, M. J. Wojcik, and S. Vogt, U. S. Patent application in progress for ANL-IN-13-092]

Compound Refractive Lenses

Outline

20 μm

50 μm

Seiboth et al, APL, 2014

"Let's drill holes into Aluminium, and use that as a lens" ...

really? yes!
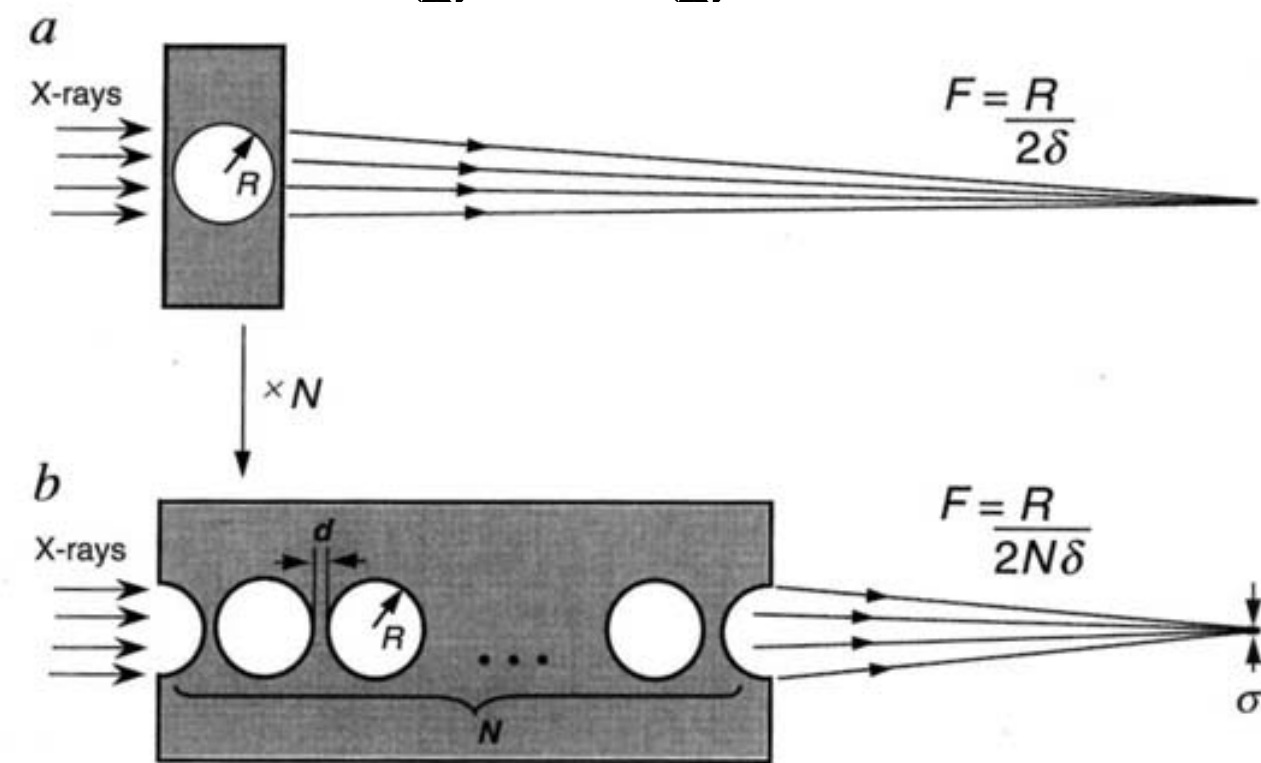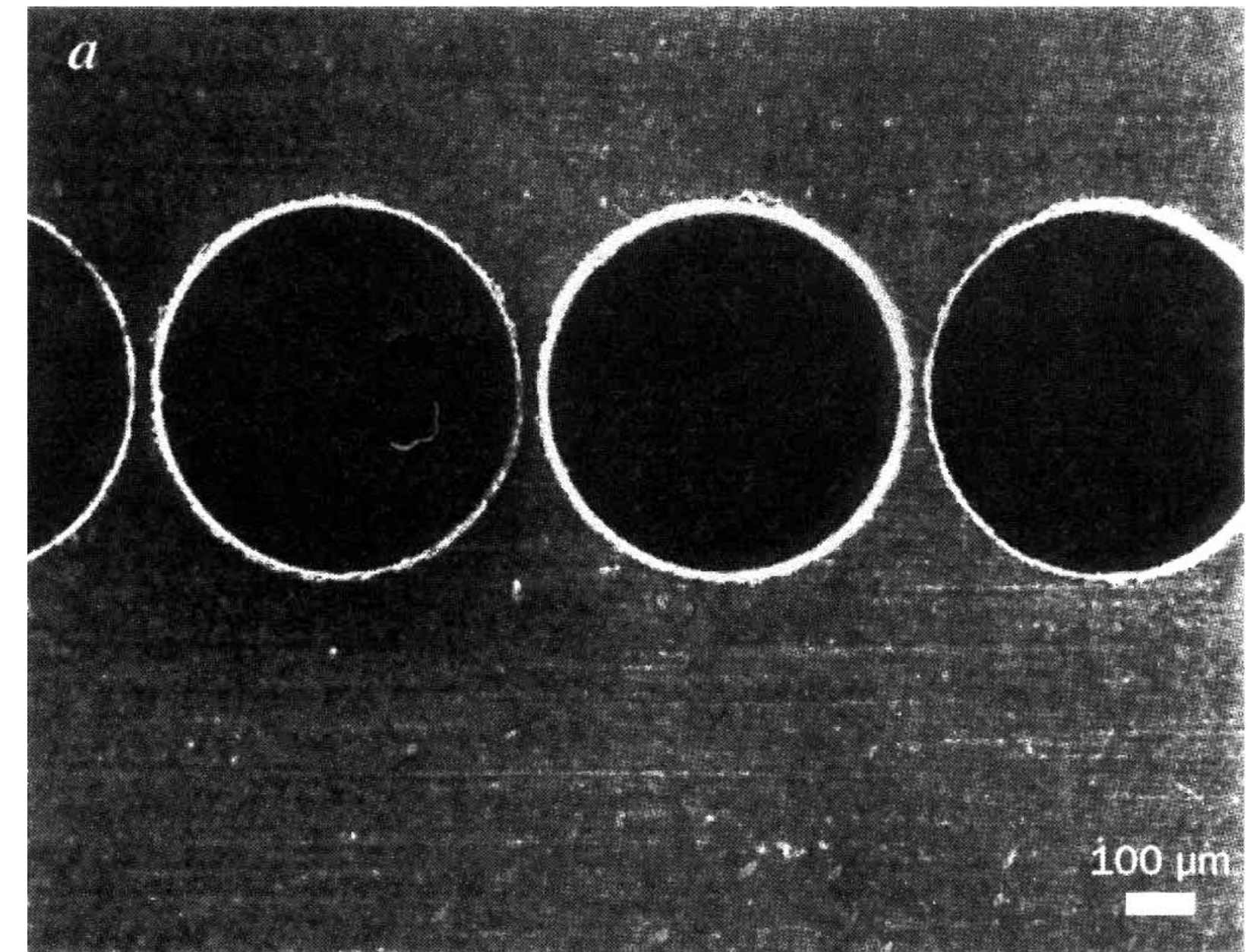

FIG. 1 Schematic diagram showing the principles of X-ray focusing by a compound refractive lens (CRL). As $(1 - \delta)$ is smaller than 1 (where δ is the decrement of the refractive index), a collecting lens for X-rays must have a concave shape. a, A simple concave lens fabricated as a cylindrical hole in the material. b, A CRL consisting of a number (N) of cylindrical holes placed close together in a row along the optical axis, focuses the X-rays at a distance that is N times shorter compared to a single lens. R is the radius of the holes, d is the spacing between the holes, λ is the X-ray wavelength, and F is the focal distance for a parallel input beam.

▶ focal length:
radius / 2 N δ

▶ with radius ~ 200 µm,
δ ~ 10^{-5}:
f ~ 10 m / N

▶ with ≥ 50 lenses,
f ≤ 0.2 m possible

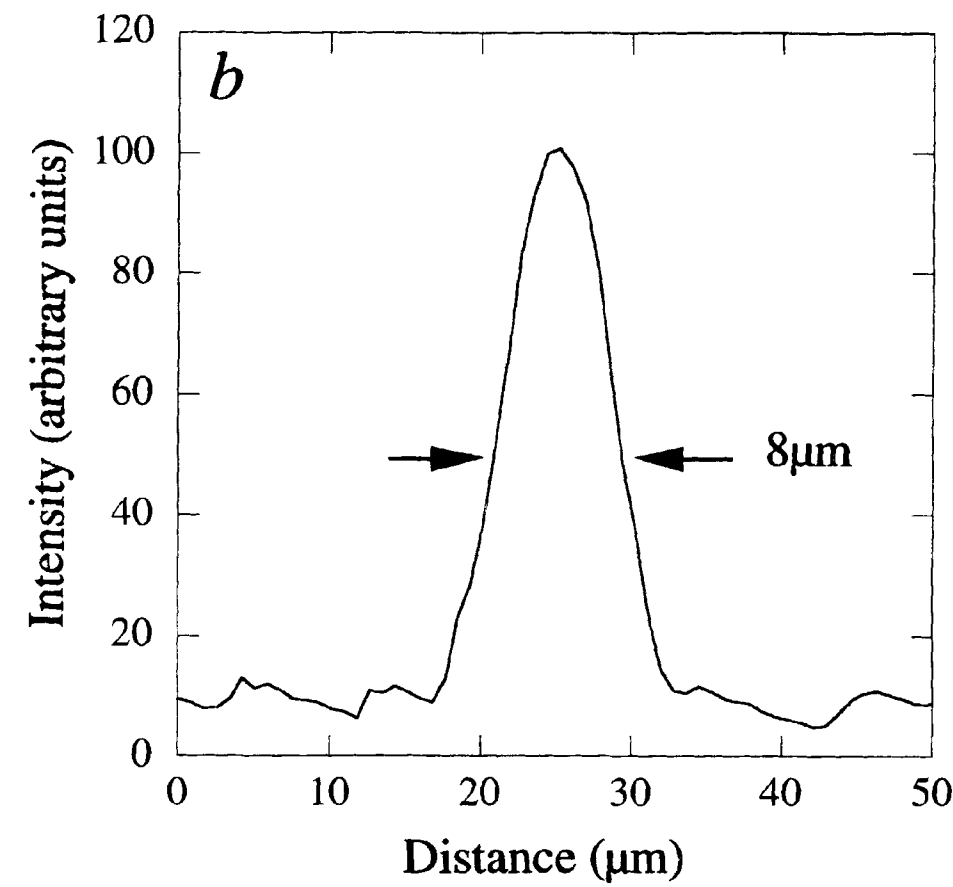Snigirev, Lengeler et al, Nature, 1996

FIG. 3 *a*, Scanning electron microscope image of the tested compound refractive lens (CRL). 30 cylindrical holes of 300-μm radius were produced in an Al–Cu alloy (with 4 wt% Cu) plate using a computer-controlled drilling machine. Spacing *d*, the minimum alloy thickness between the holes, was ~25 μm. The length of the CRL was 19 mm. *b*, Measured profile of the focused X-ray beam taken at 1.8 m from the CRL. The width of the focal line is ~8 μm, which almost corresponds to the source size of 150 μm reduced by a demagnification factor $r_0/r_f = 20$.

▶ 30 holes

▶ 300 μm radius each

▶ X-ray energy: 14 keV

▶ focal length: 1.8 m

▶ line focus: 8 μm

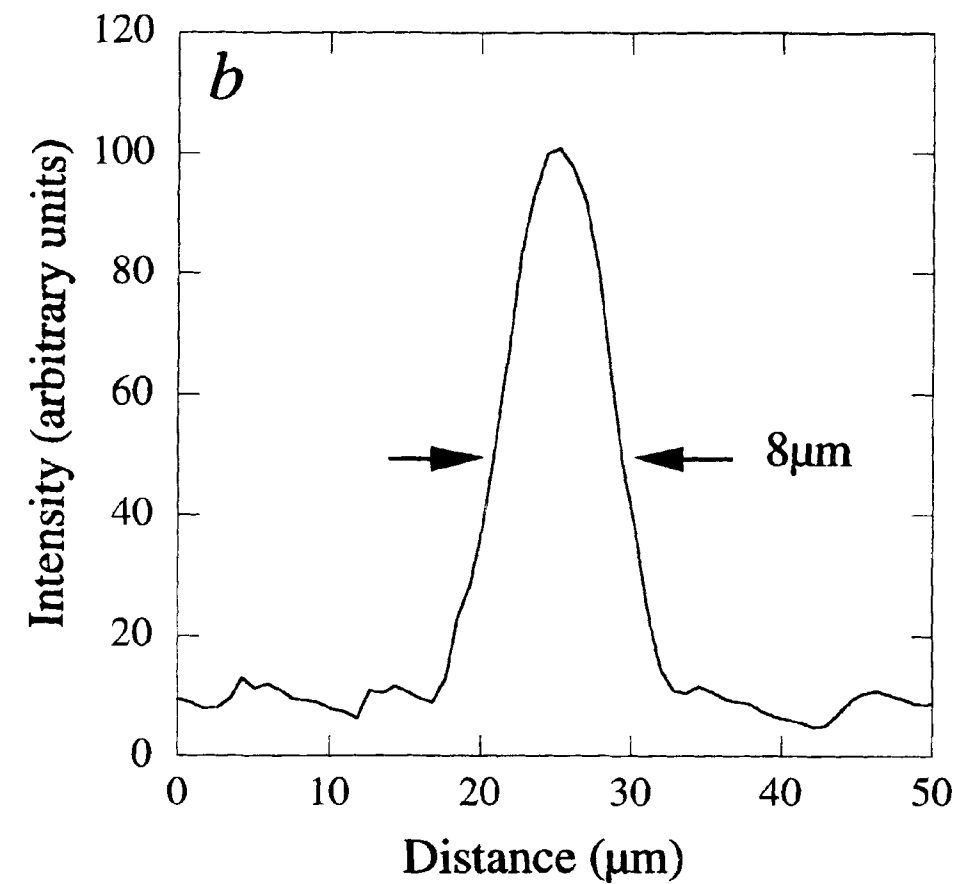Snigirev, Lengeler et al, Nature, 1996

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▶ 300 µm radius each

▶ X-ray energy: 14 keV

▶ focal length: 1.8 m

▶ line focus: 8 µm

questions for following decades:

▶ better fabrication?

▶ best material?

▶ nanometre possible?

▶ flexibility?

If no-one has done it, maybe it's not possible.
If no-one has tried yet, maybe it is possible.

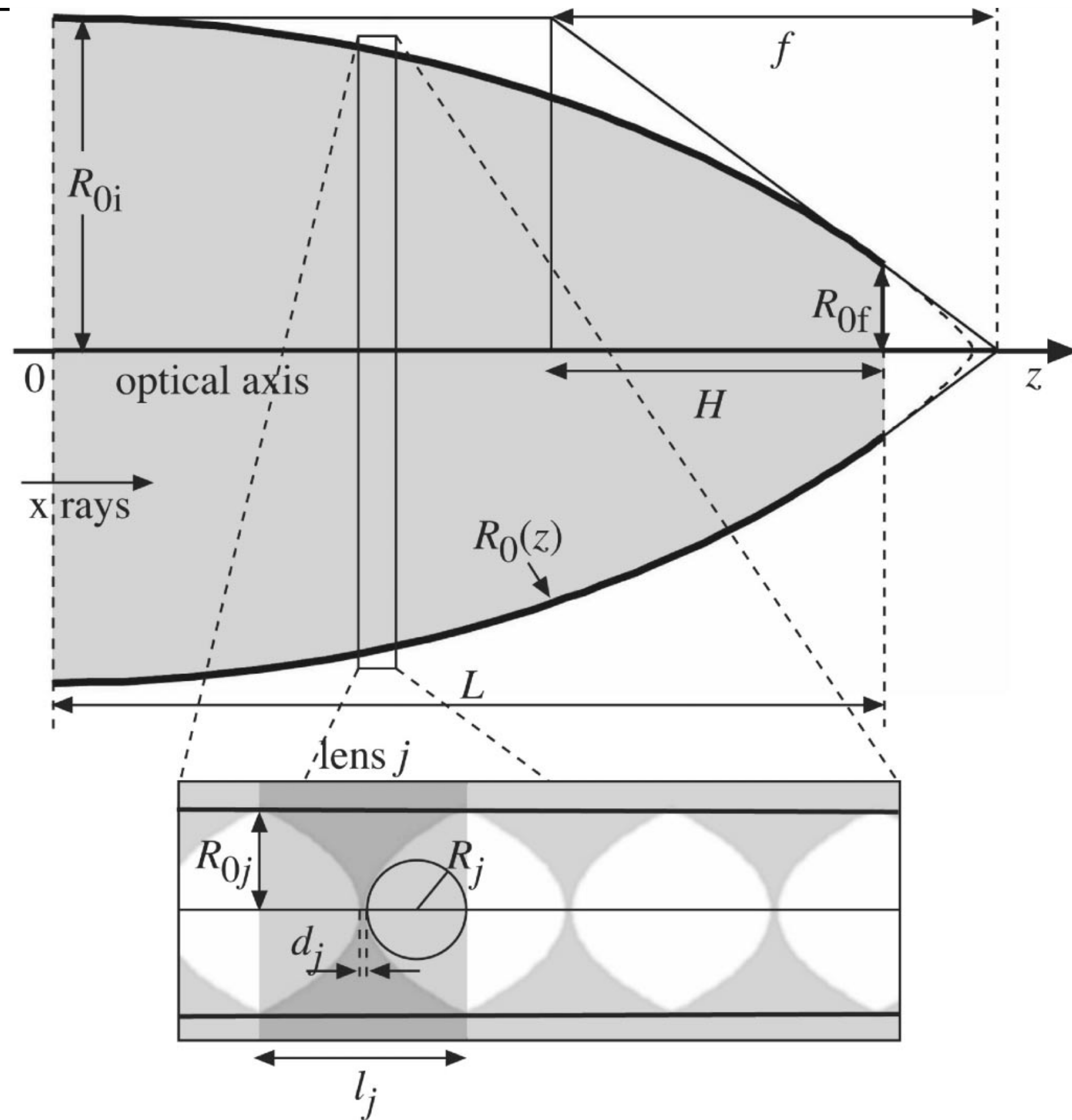Snigirev, Lengeler et al, Nature, 1996

Adiabatic Lenses

FIG. 1. Adiabatically focusing x-ray lens. The lens is composed of a large number of individual (aspherical) refractive lenses, whose aperture is matched to the converging beam size, increasing the refractive power per unit length along the lens.

Schroer, Lengeler: PRL, 2005
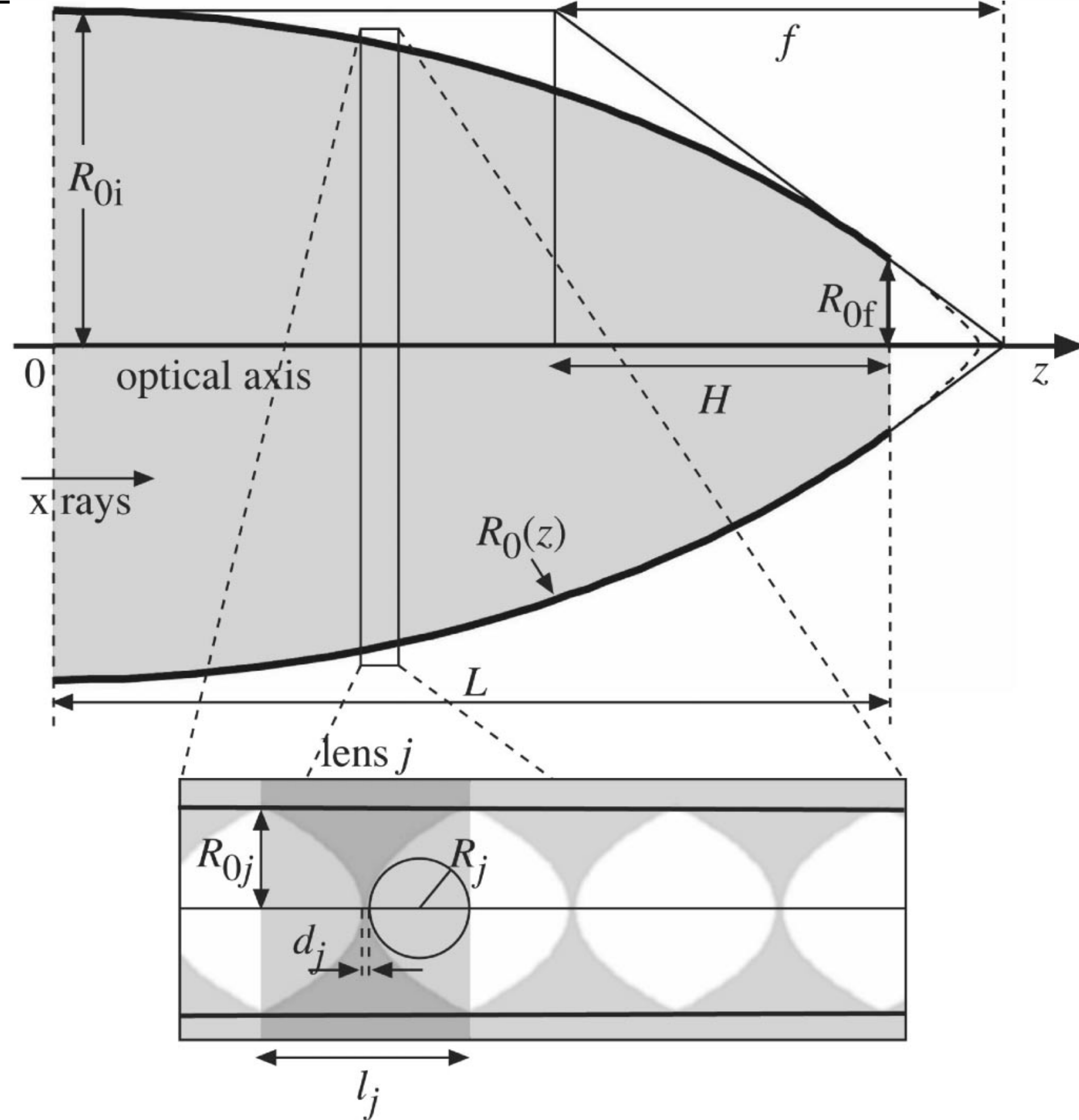
Adiabatic Lenses

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where $\mu' = (1 - d_j/l_j)\mu$ and μ is the linear attenuation coefficient. Using Eqs. (4) and (5), the numerical aperture $\text{NA} = D_{\text{eff}}/(2f)$ for a distant source is

$$\text{NA} = \sqrt{\delta'}\sqrt{4\frac{a}{R_{0i}}\left[1 - \exp\left(-\frac{R_{0i}}{a}\right)\right]\log\frac{R_{0i}}{R_{0f}}}, \quad (6)$$

where $a = \frac{4\sqrt{\delta'}}{\sqrt{\pi}\mu'}$ is a material specific characteristic aperture shown in Fig. 2 for different materials. A larger a yields a larger numerical aperture, favoring materials with low atomic number Z. In addition, the numerical aperture is proportional to $\sqrt{\delta'}$, favoring lens materials with large mass density ρ. The largest numerical apertures are therefore expected for high density low Z materials, such as diamond.
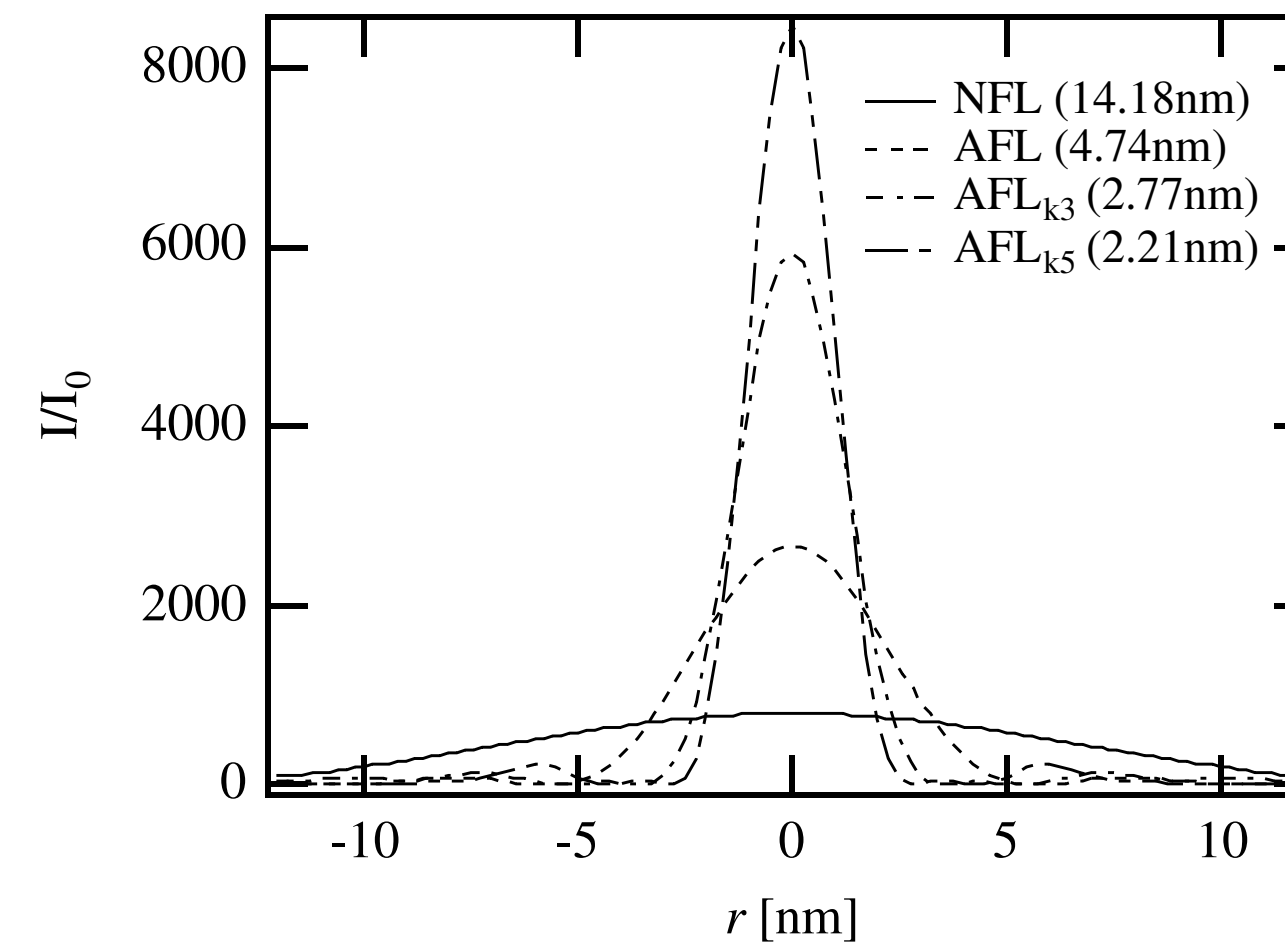

FIG. 5. Lateral beam profiles for a nanofocusing lens, an adiabatically focusing lens, and two kinoform AFLs with three and five segments, respectively.

Schroer, Lengeler: PRL, 2005
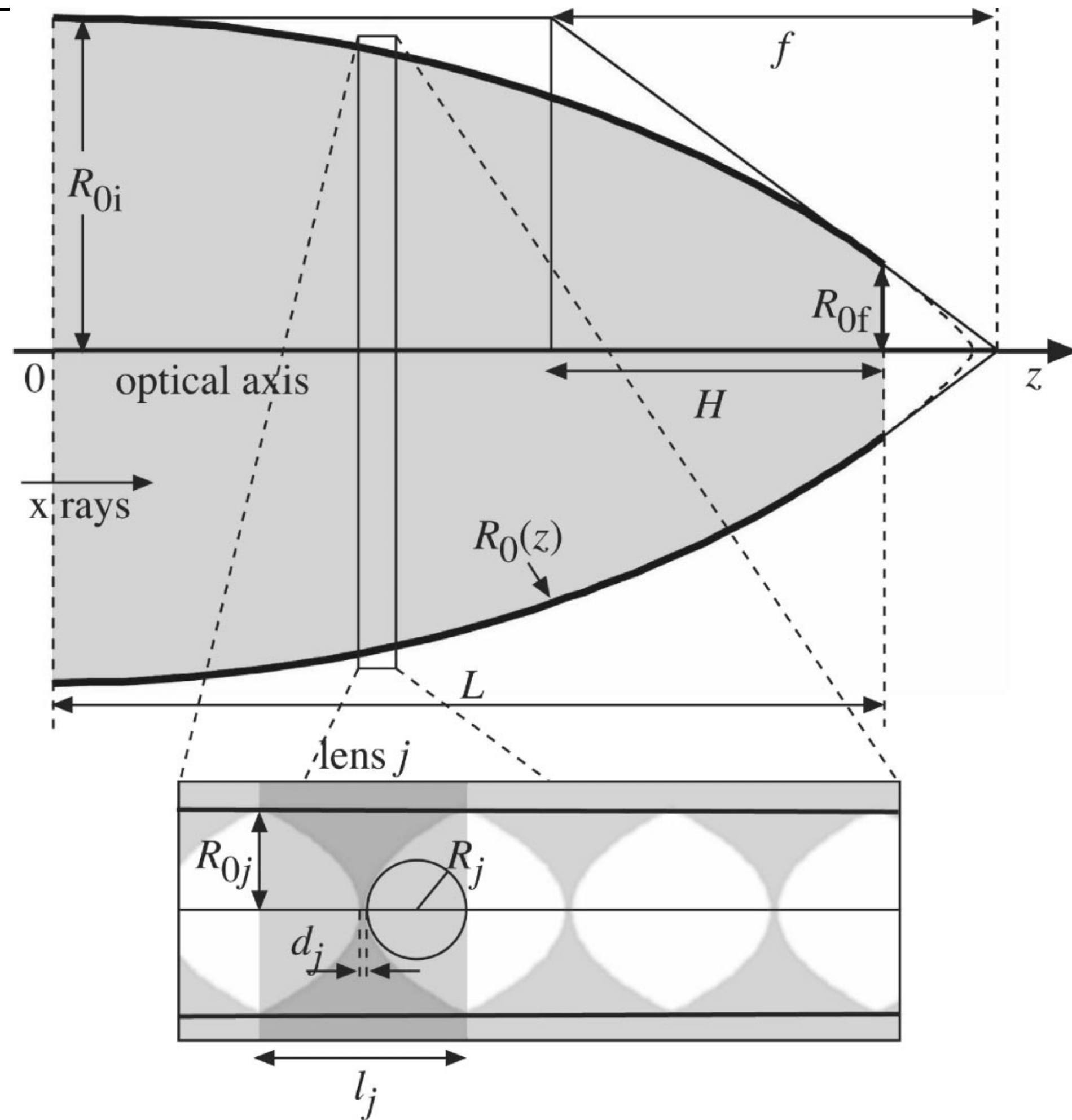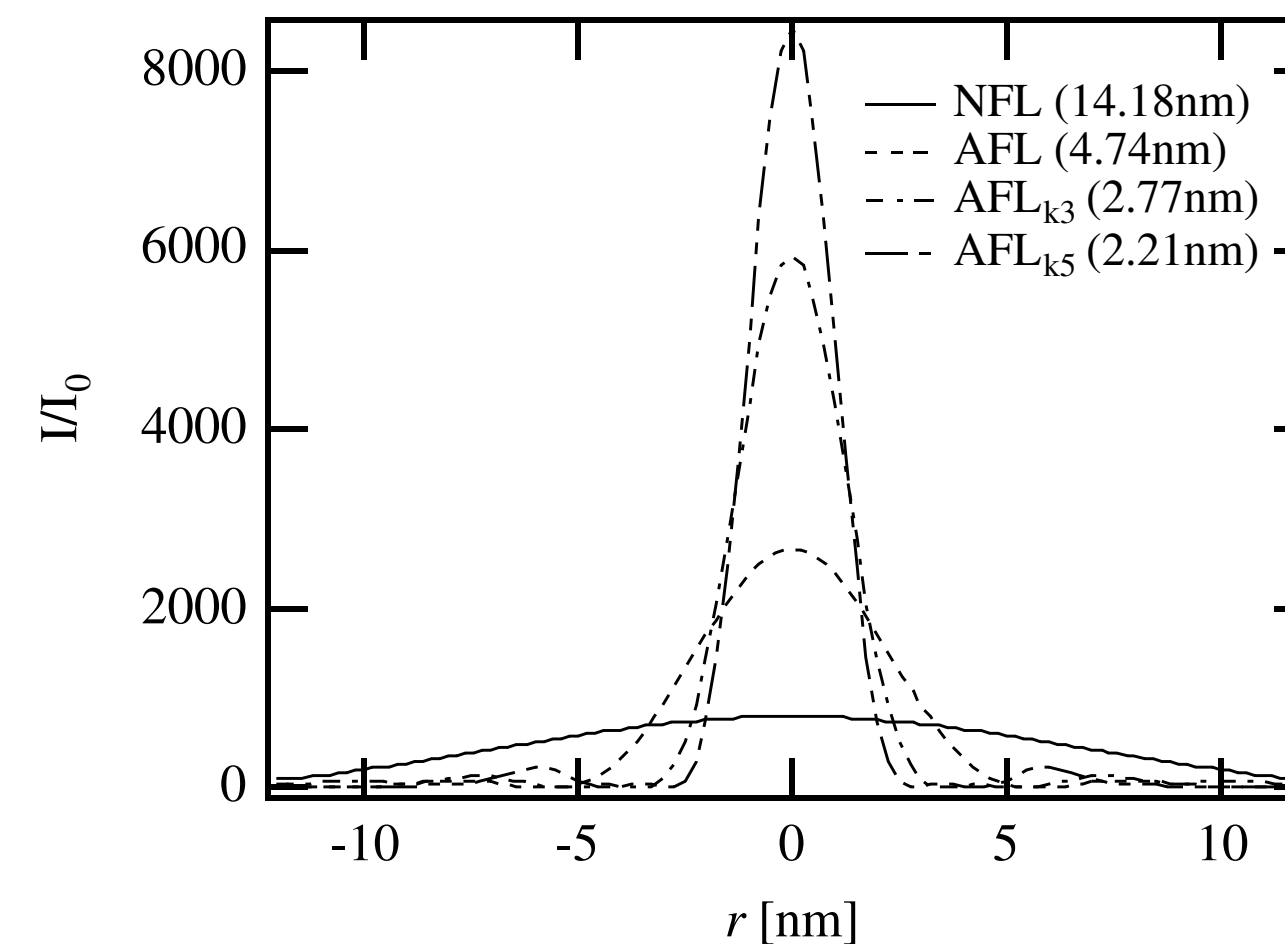
Adiabatic Lenses

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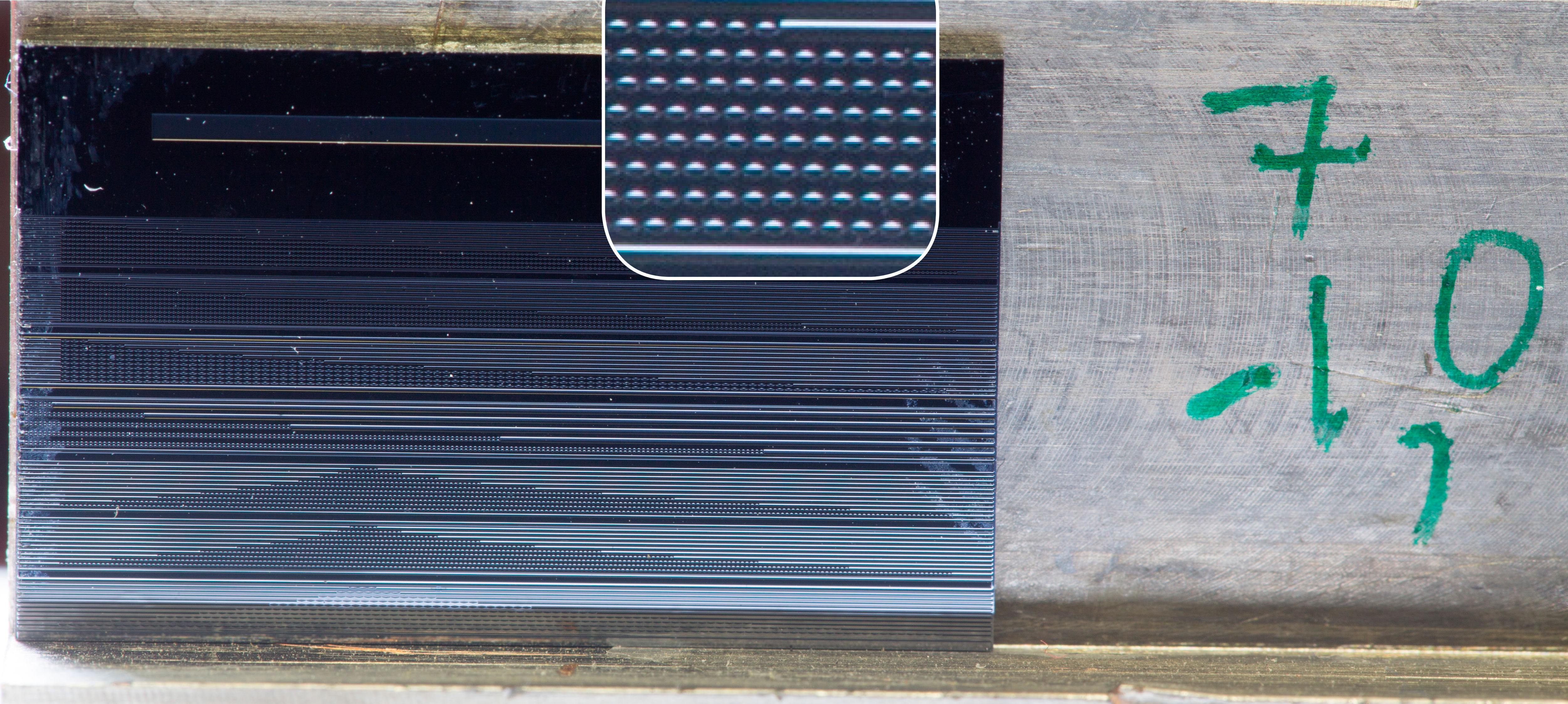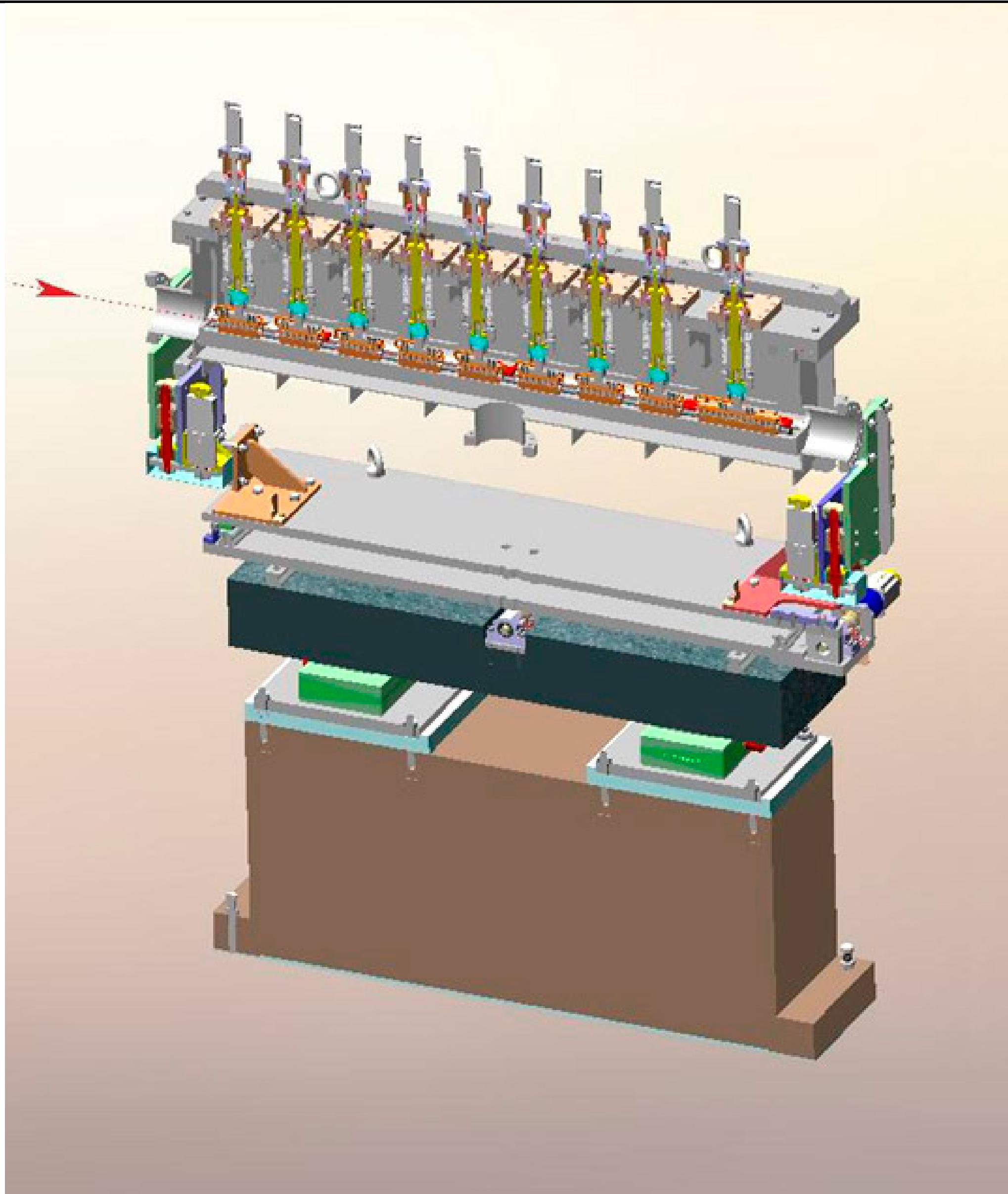

FIG. 5. Lateral beam profiles for a nanofocusing lens, an adiabatically focusing lens, and two kinoform AFLs with three and five segments, respectively.

- ▶ absorption limit, use low-Z (Be, Al, C)
- ▶ need phase shift, use high mass density e.g., diamond
- ▶ adapt lens diameter to local beam size
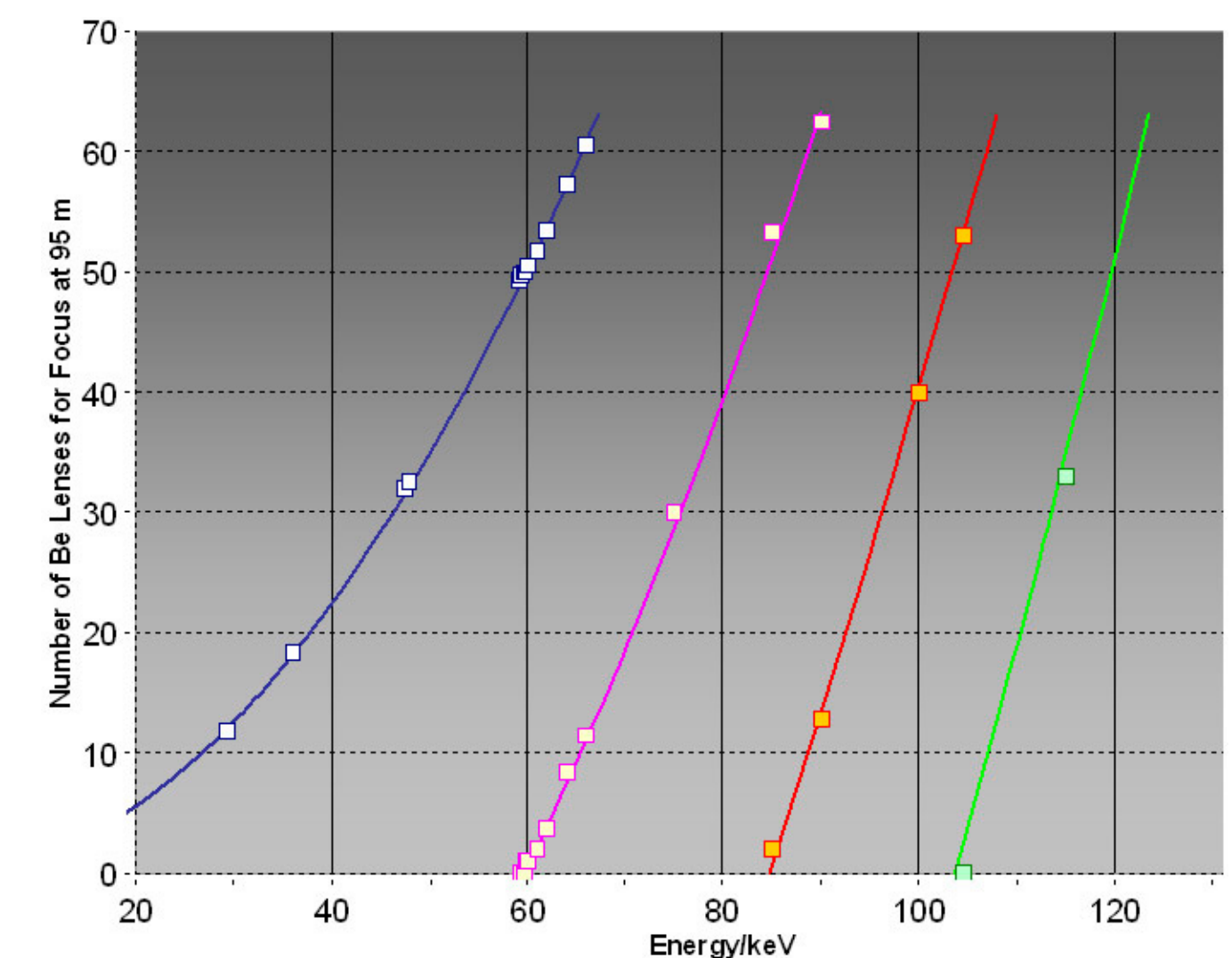- ▶ aspherical holes to improve quality
- ▶ sub-10 nm possible

Schroer, Lengeler: PRL, 2005

Transfocators

► zoom capabilities:
cartridges of 2^n lenses

► choose combination based on
x-ray energy – focal length – beam size

► developed at ESRF, used at many synchrotrons

► especially at higher energies, 20 … 100 … keV
spot size in micro metre range

Vaughan, Snigirev, Wright et al

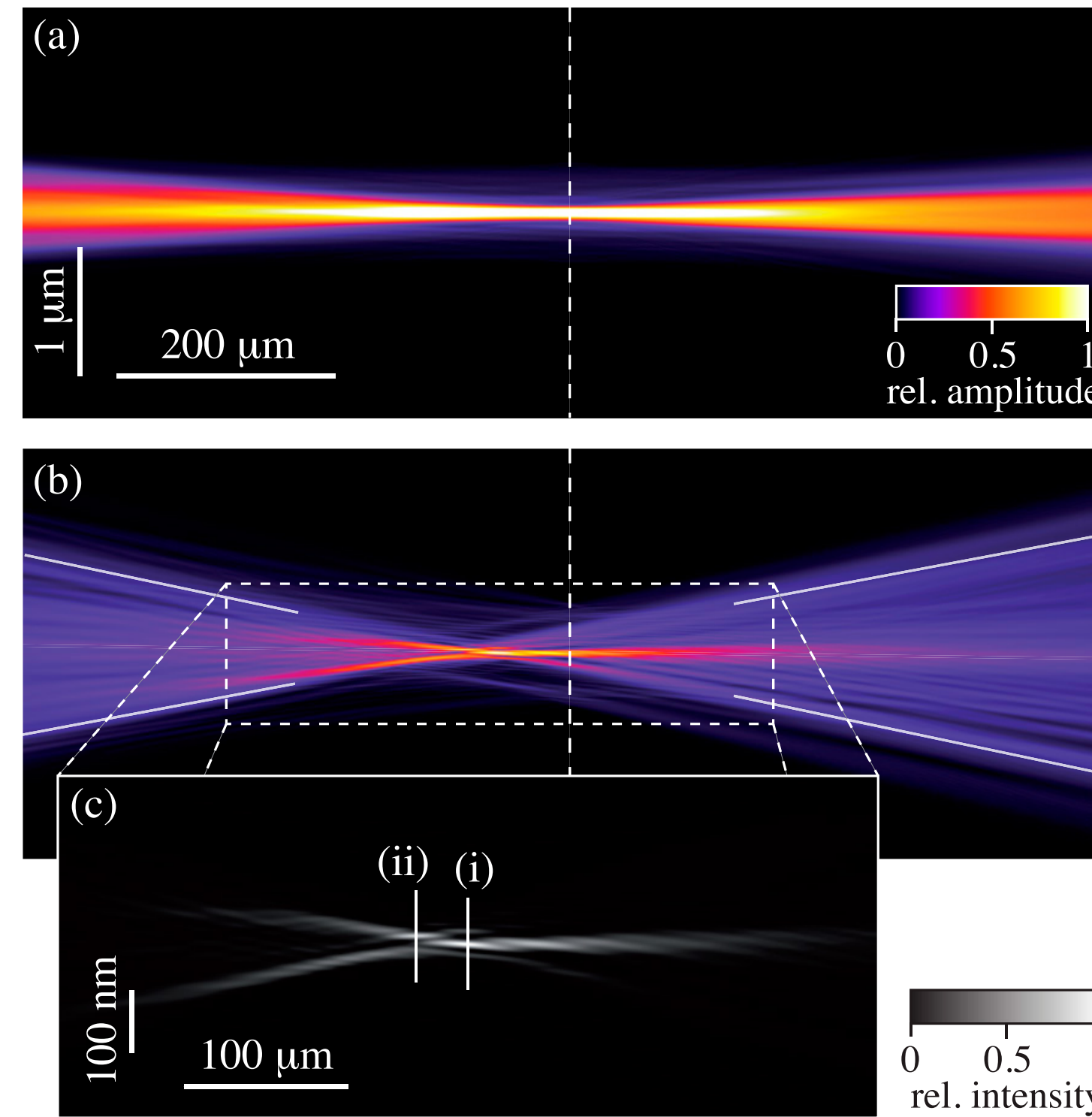*Focusing hard x rays beyond the critical angle of total reflection by adiabatically focusing lenses @ 20 keV*
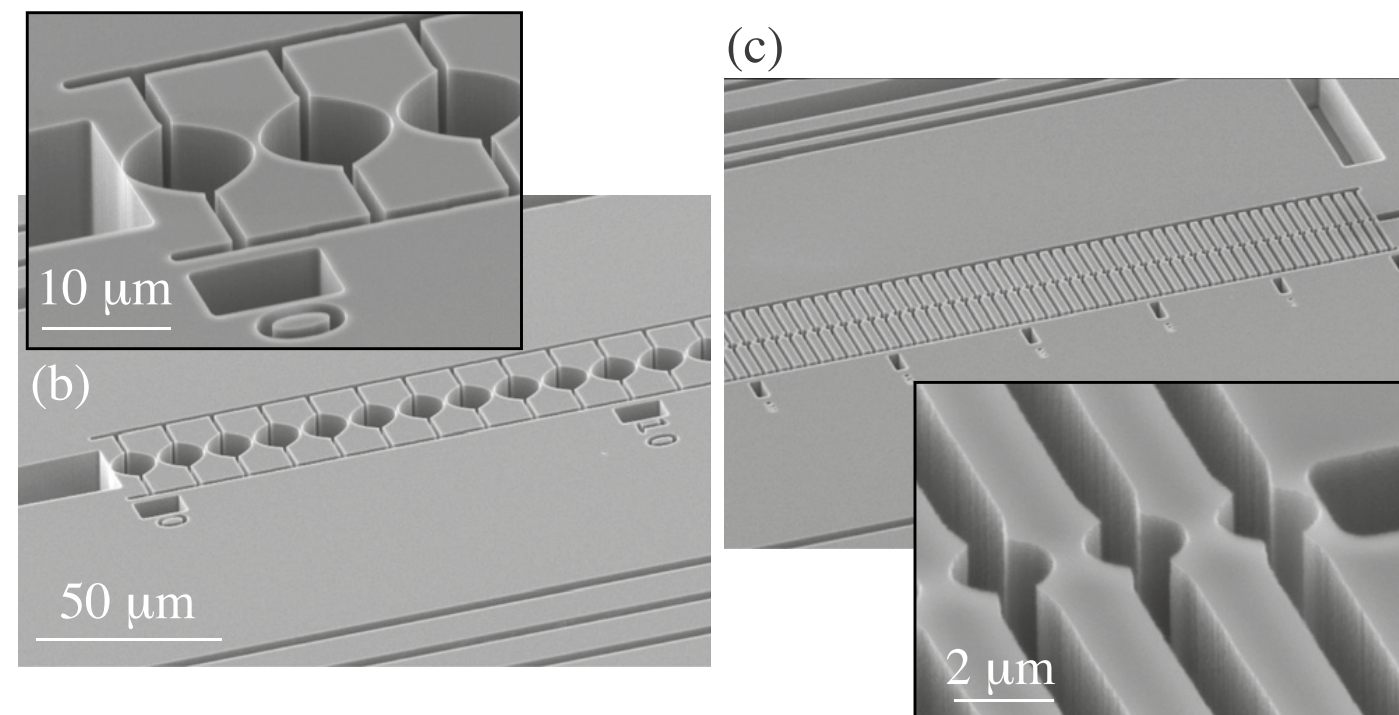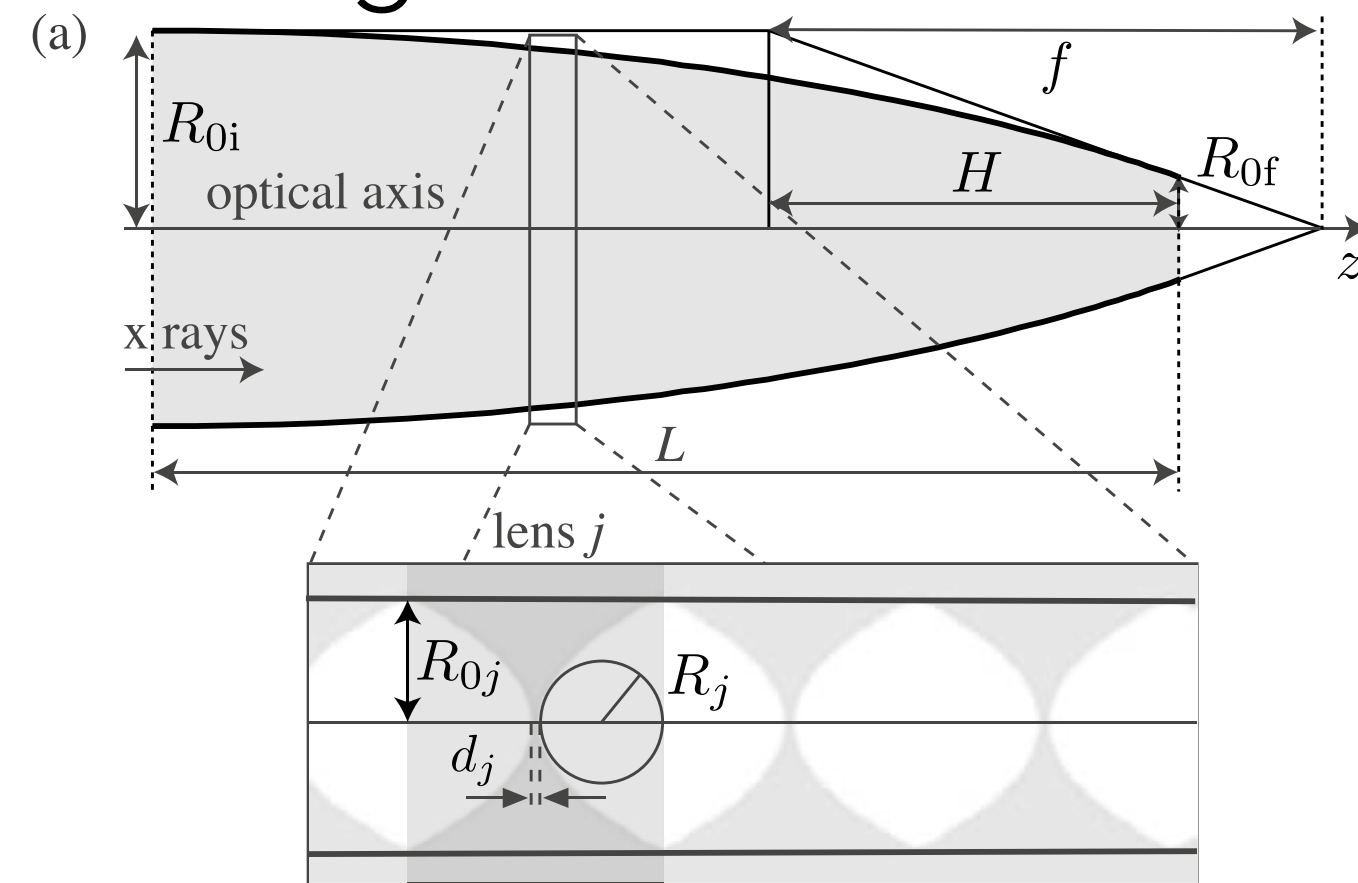

FIG. 3. Caustic of the nanofocused beam: amplitude of the wave field projected onto (a) the vertical and (b) horizontal plane. The dashed line in (a) and (b) depicts the sample plane, in which the ptychogram was recorded. The semi-transparent lines in (b) delineate the numerical aperture. (c) Intensity distribution around the focus [dashed area in (b)] projected onto the horizontal plane. Line (i) in (c) is the section with the highest peak intensity (focus), line (ii) is a section through the laterally smallest speckle.

The horizontal focus that is generated by the AFL is shown in Figs. 3(b) and 3(c). Ideally, the beam would be similarly gaussian as in Fig. 3(a) but with a significantly higher numerical aperture. From the reconstruction of the caustic, it becomes apparent that the AFL suffers from aberrations, generating a far from ideal focus. The brightest speckle [Fig. 3(c)] can be considered the focus and lies in the plane marked by (i) in Fig. 3(c). The horizontal intensity profile in this plane is shown in Fig. 4(i) and shows a focal spot size of 18.4 nm (FWHM). About 65% of the radiation fall into this
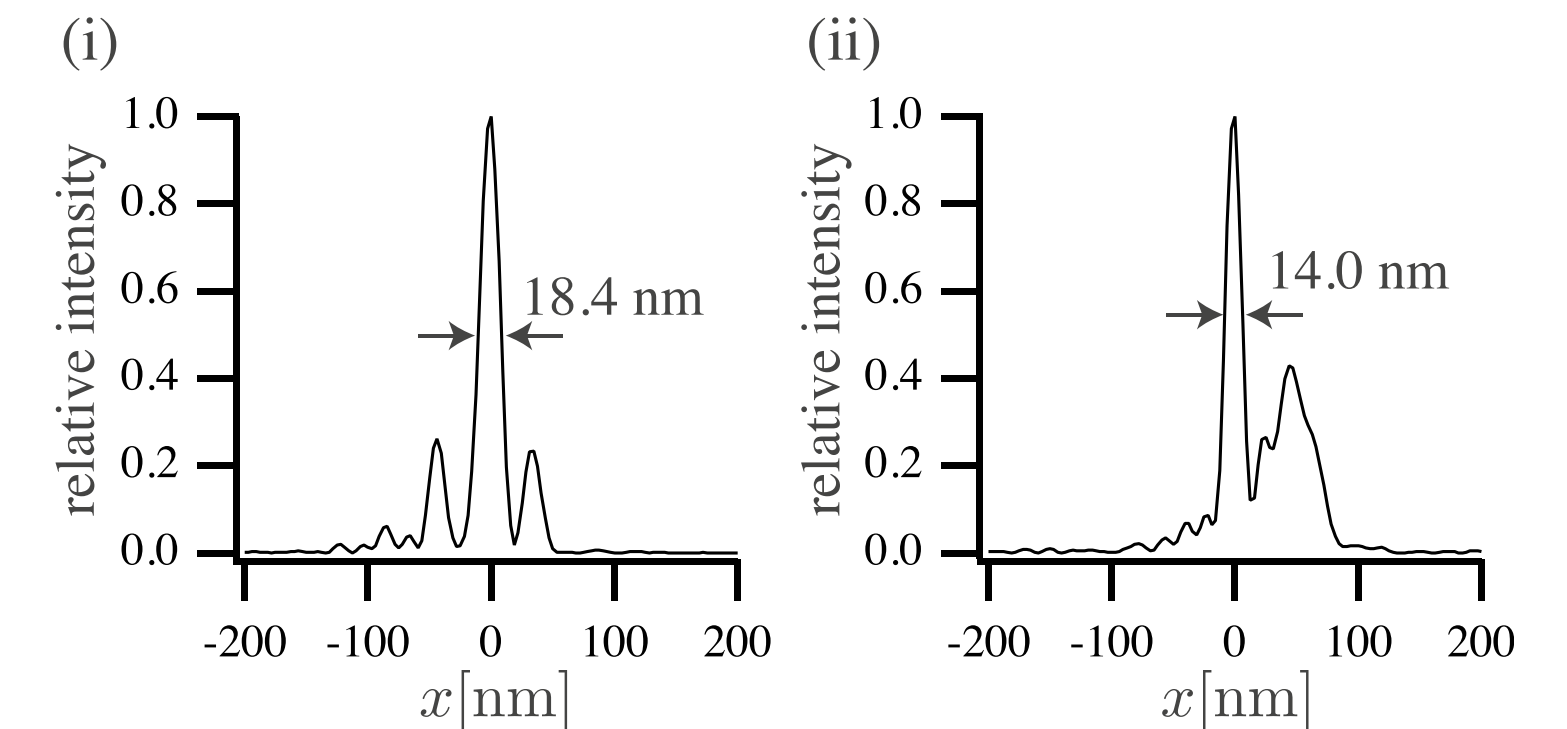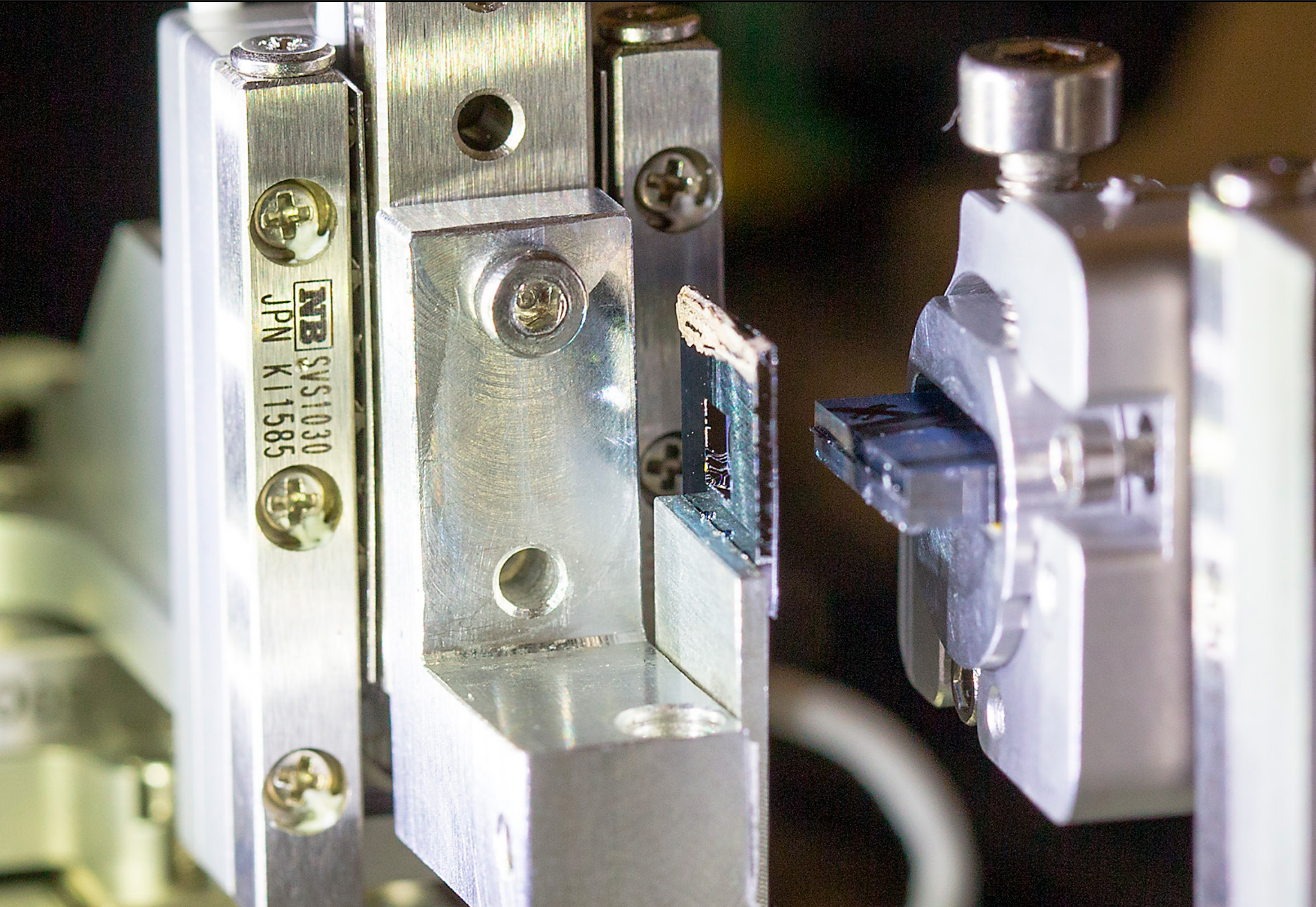

FIG. 4. (i) Horizontal intensity distribution in the focal plane of the AFL [line (i) in Fig. 3(c)]. The central speckle has a lateral size of 18.4 nm (FWHM). (ii) horizontal intensity distribution at the section with the laterally smallest speckle [line (ii) in Fig. 3(c)]. The smallest speckle has a lateral size of 14.0 nm (FWHM).
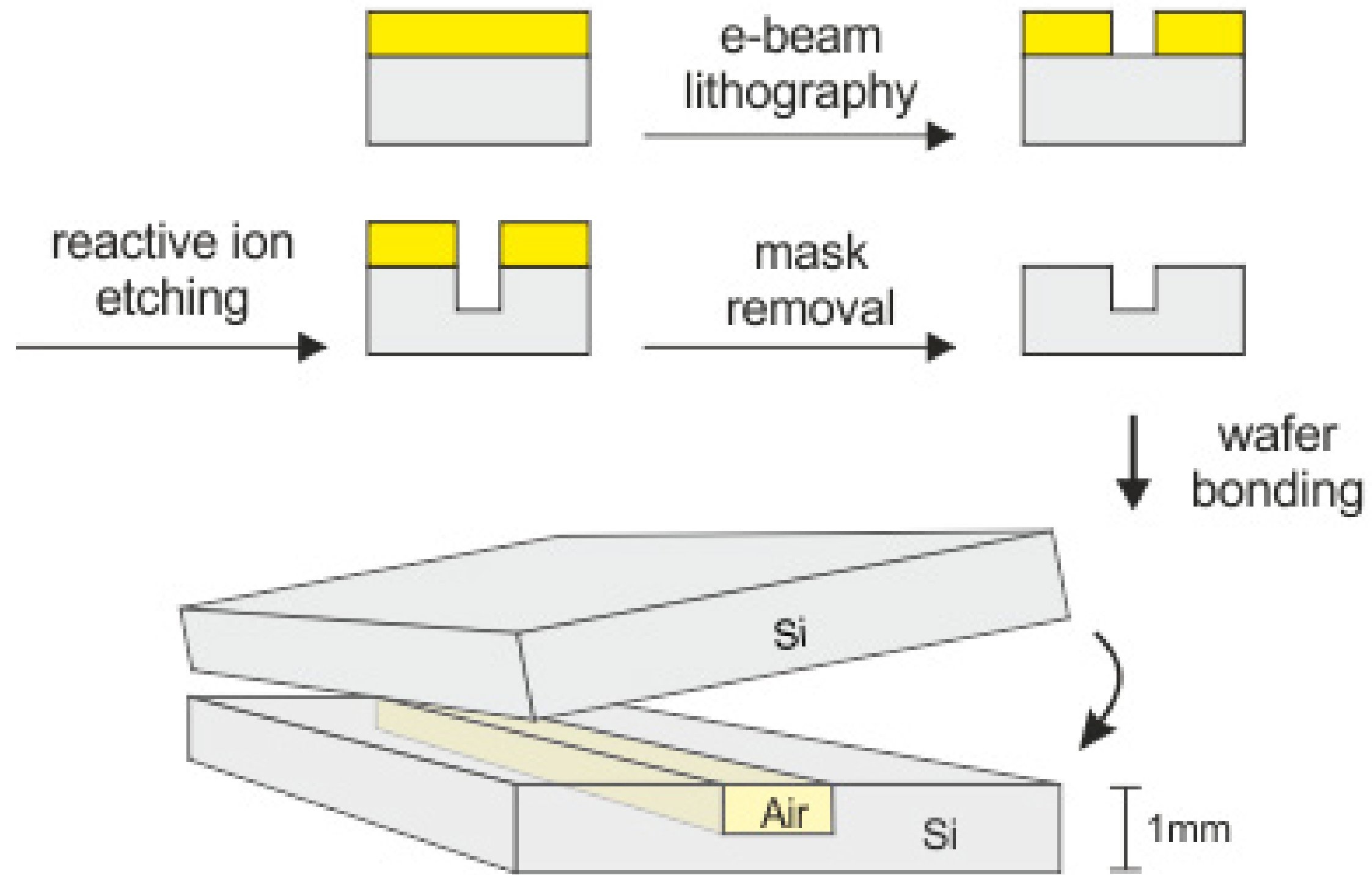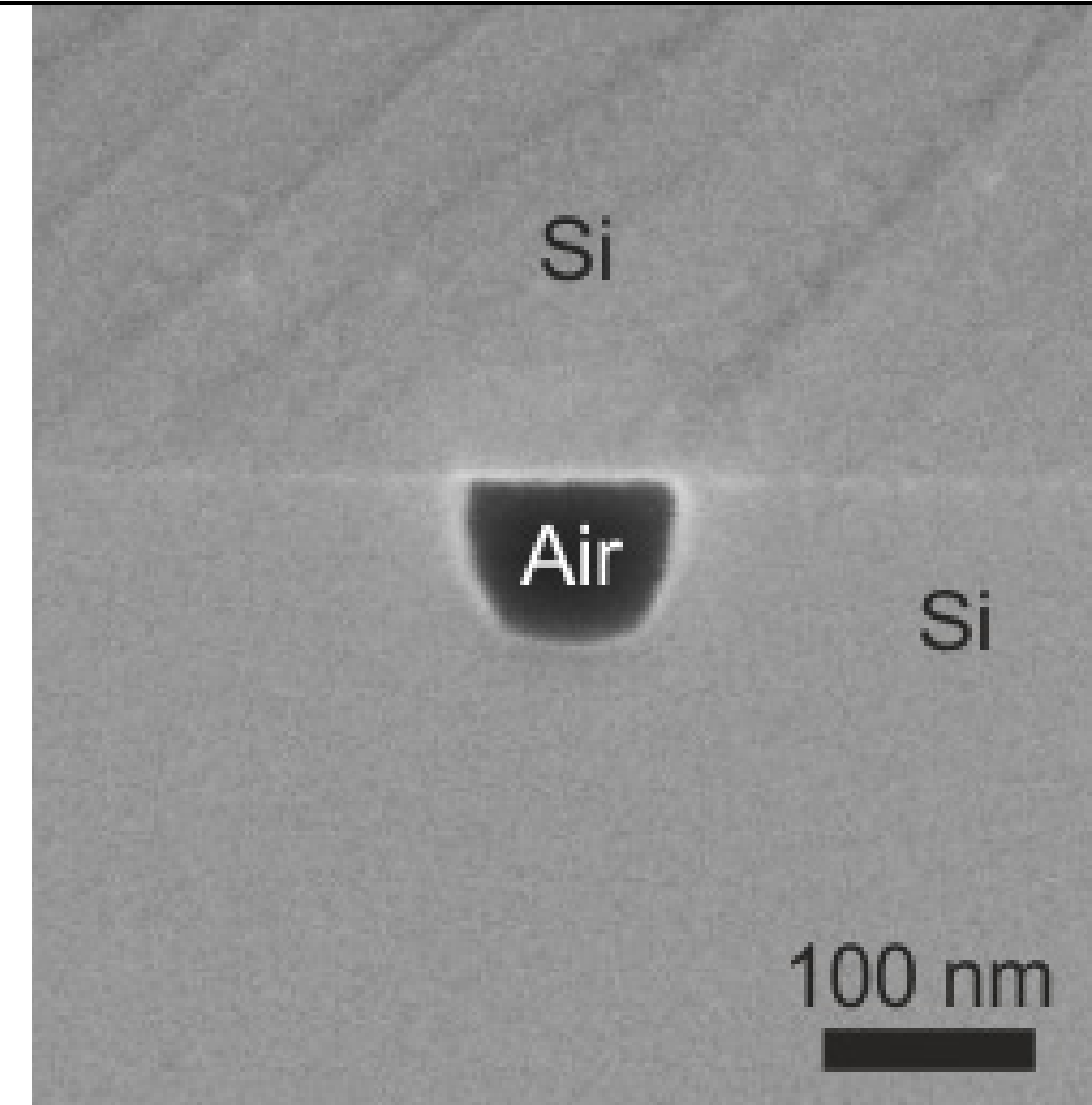
Patommel, Schroer et al, APL, 2017

Waveguides
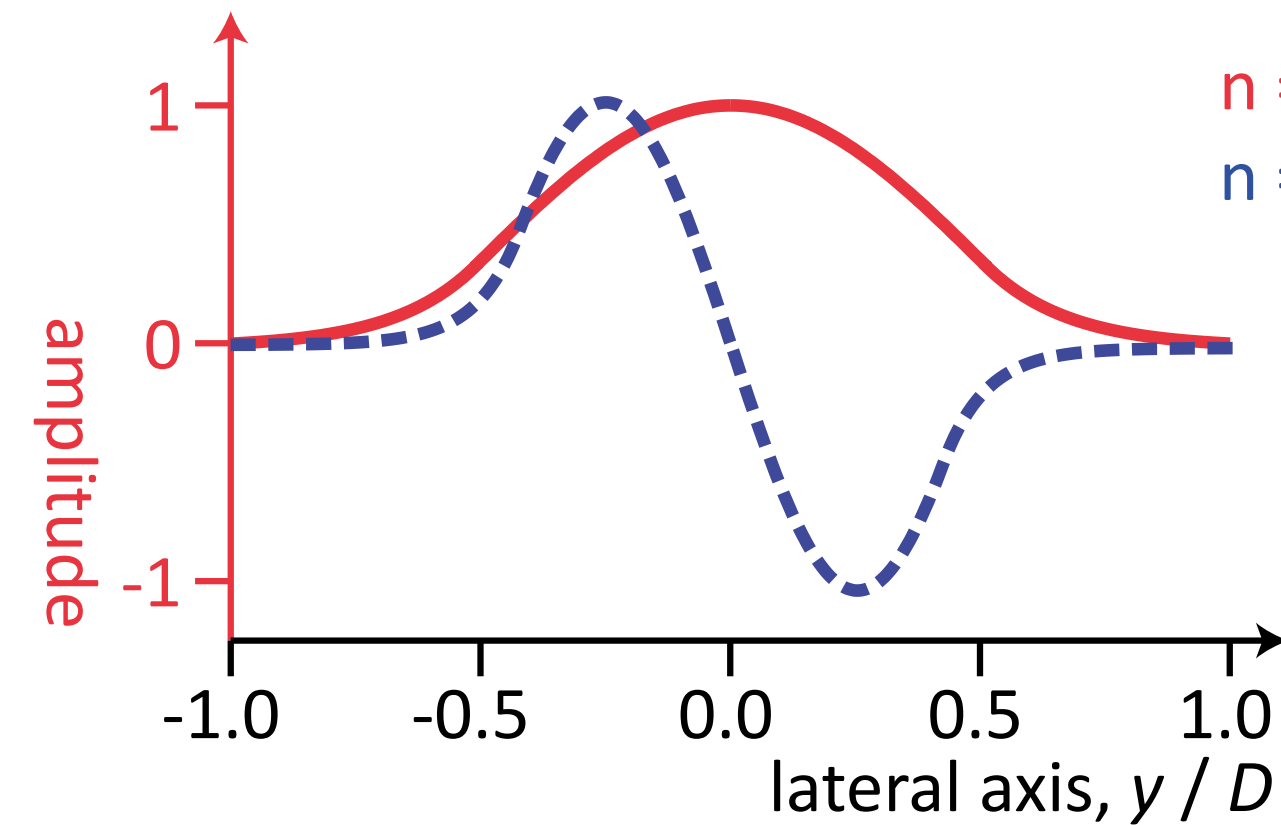
▶ rays reflected back and forth, or:

▶ wave trapped inside cavity

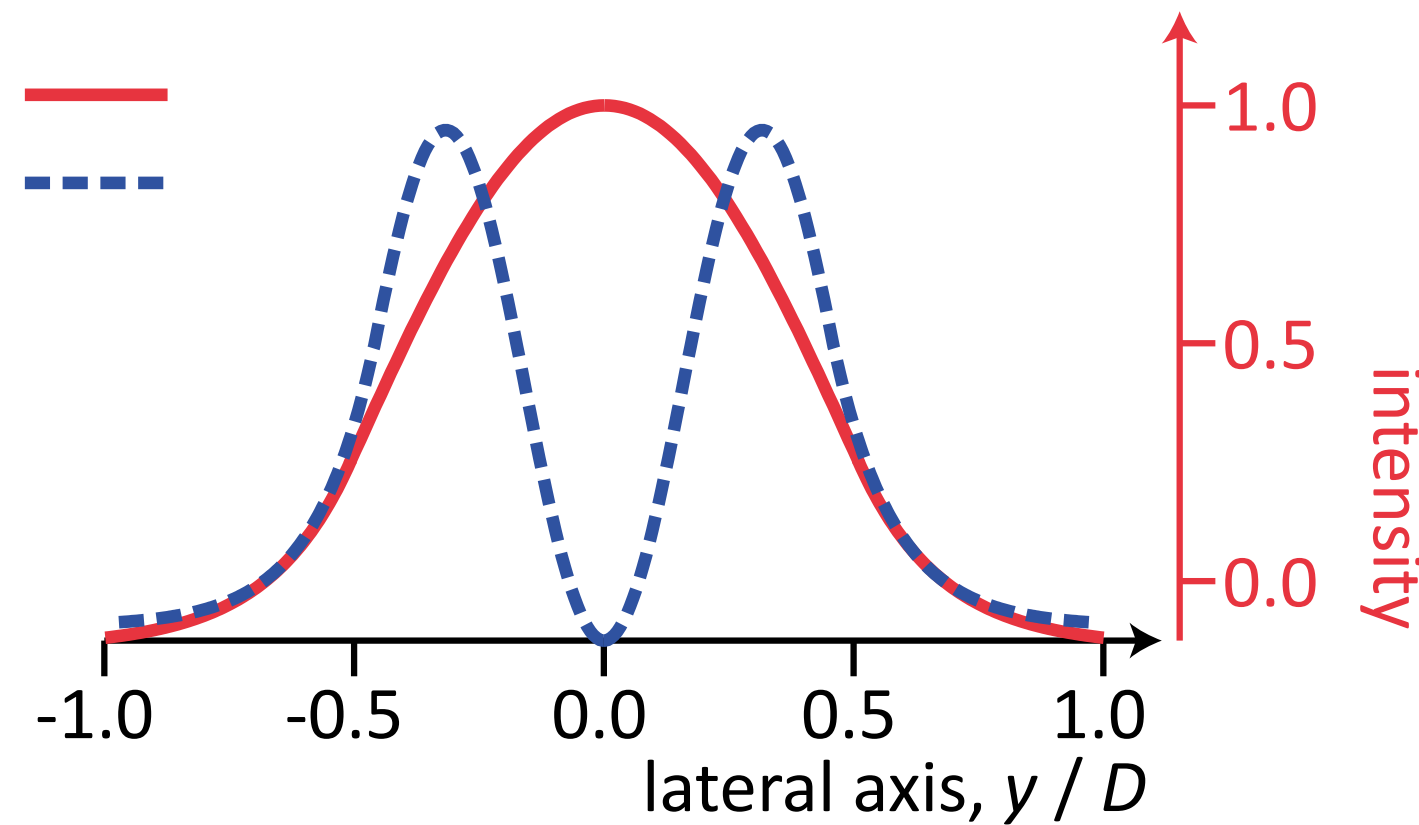▶ channel size: ~ $(20\,\text{nm})^2$

▶ channel length: ~ 1 mm

Analytical and Numerical WG Design

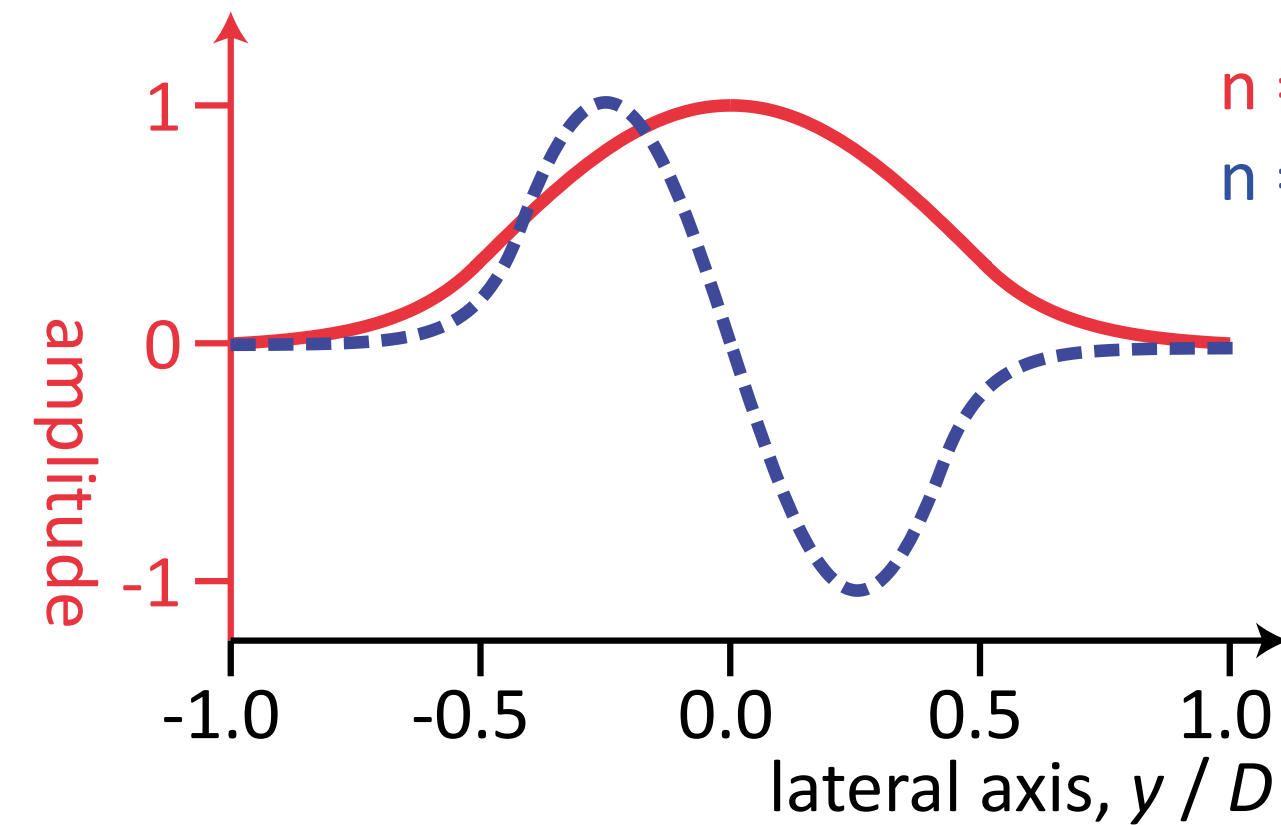**(a)** Waveguide modes - amplitude

(b) Waveguide modes - intensity

► first and second guided mode,
solving eigenvalue equation in cavity

► for Si: one mode per 20 nm channel width

► understand WG fundamentals, including
curvature, real structure, multiple components

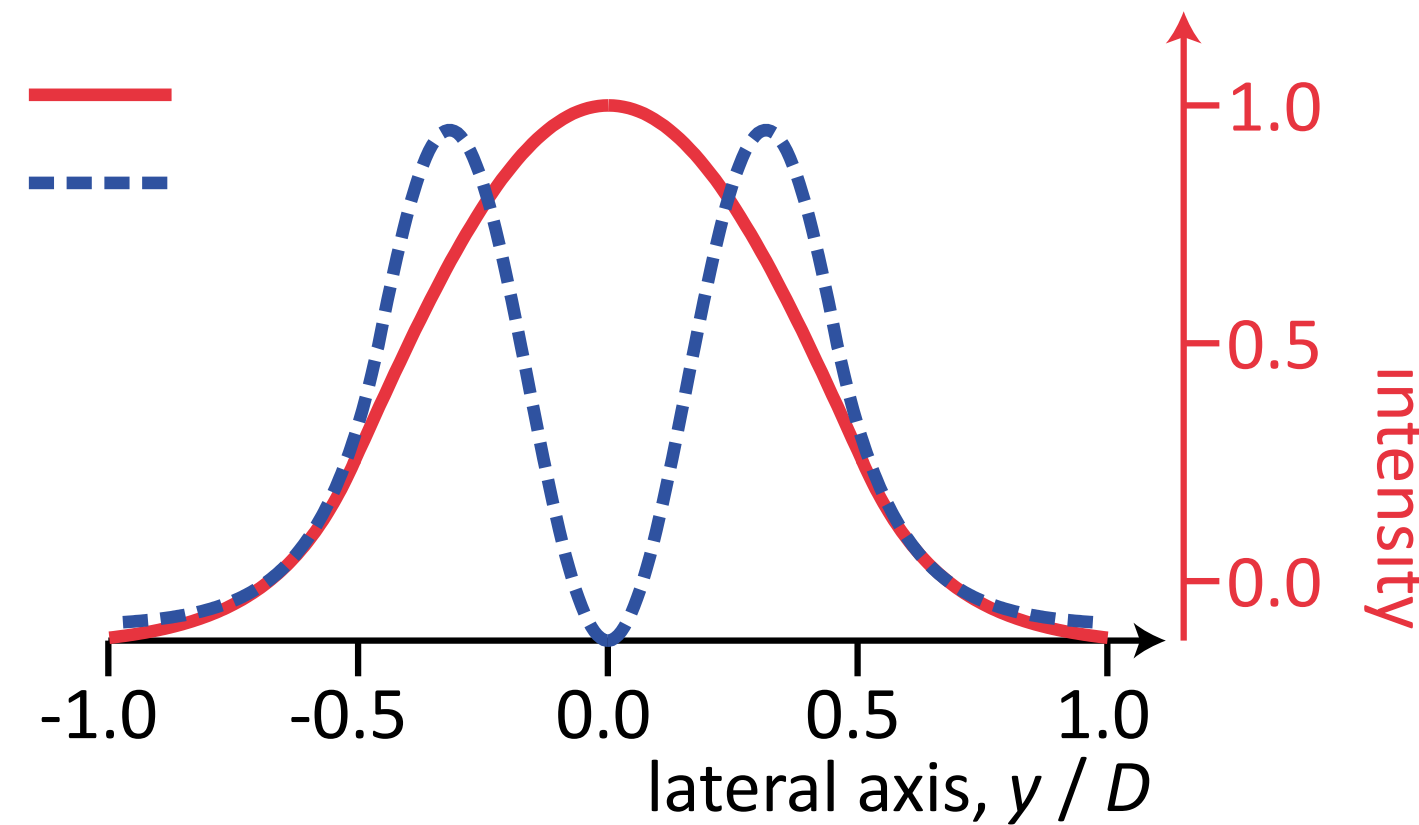# Analytical and Numerical WG Design
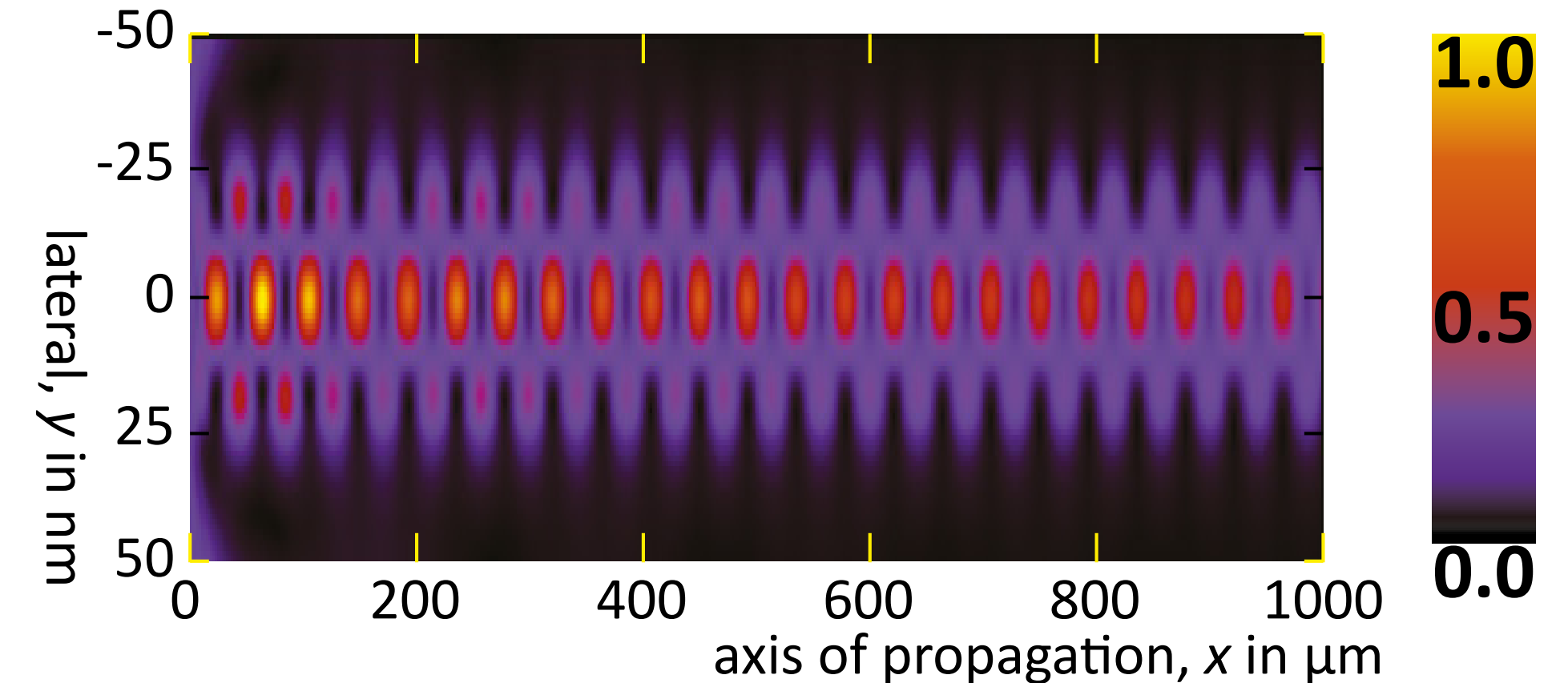
(a) Waveguide modes - amplitude

(b) Waveguide modes - intensity
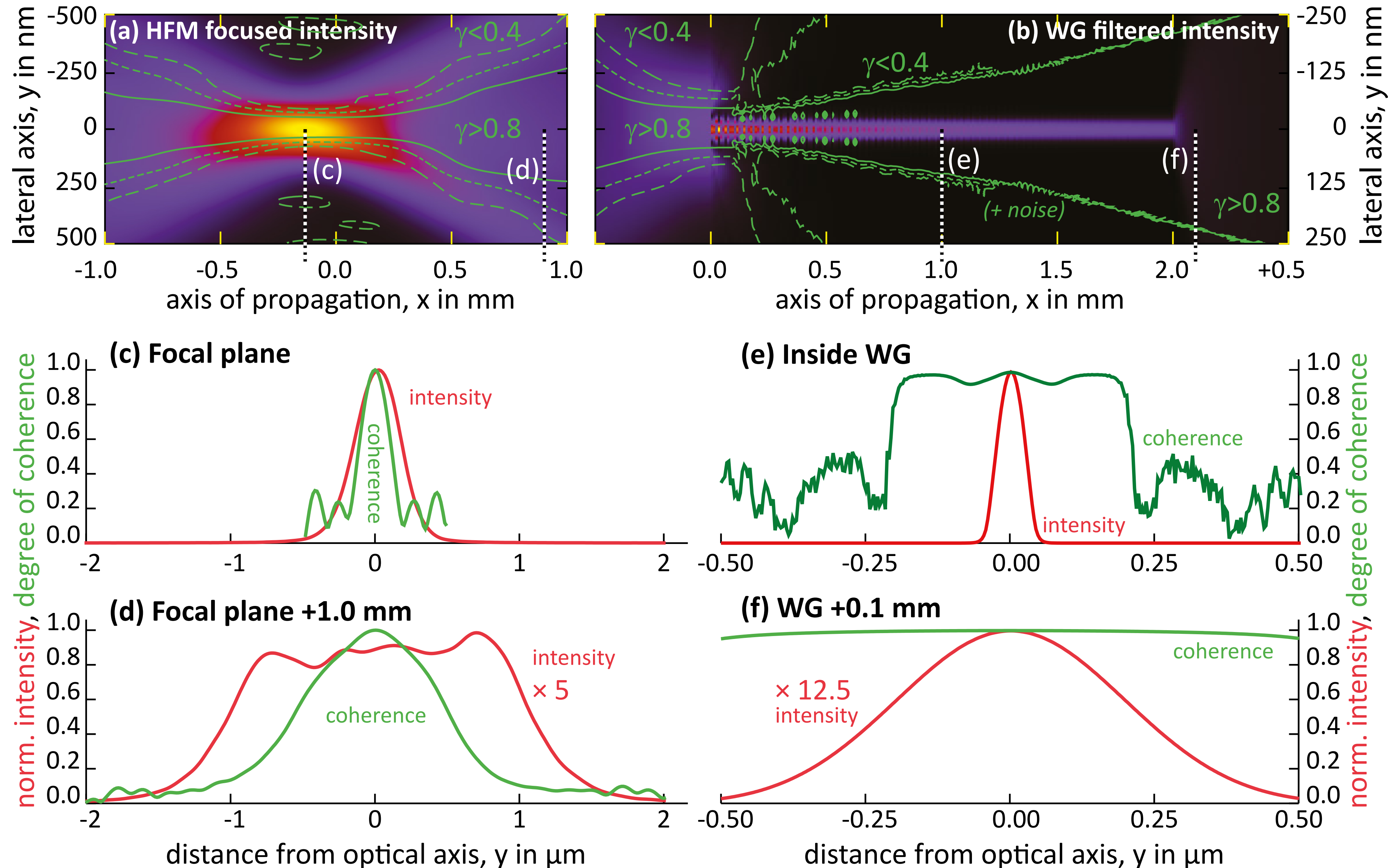

(a) Intensity distribution, $\theta = 0$

► first and second guided mode,
   solving eigenvalue equation in cavity

► for Si: one mode per 20 nm channel width

► understand WG fundamentals, including
   curvature, real structure, multiple components

► simulation inside WG,
   solving parabolic wave equation

► mode beating of 1st and 3rd

► design WGs, including
   bending, curving and splitting

Coherence Filtering

(a) HFM focused intensity

(b) WG filtered intensity

(c) Focal plane

(d) Focal plane +1.0 mm

(e) Inside WG

(f) WG +0.1 mm

► WG is placed in pre-focus, e.g. KB

► illumination is only of partial coherence

► WG accepts discrete modes, which are fully coherent

► further filtering by absorption of higher modes

► in addition, beam is cleaned from KB stripes
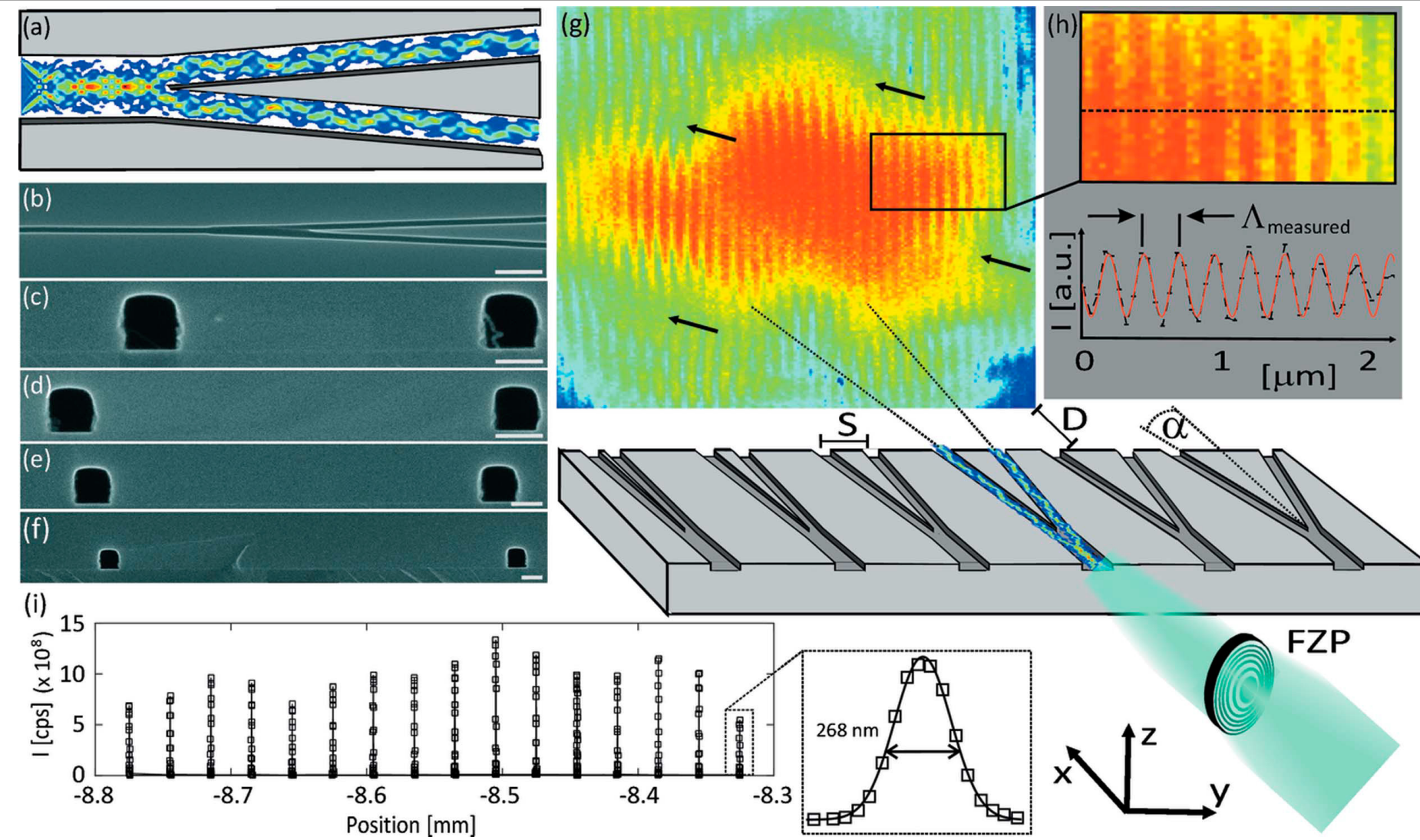
Osterhoff, PhD thesis, 2011

Figure 1

Experimental geometry. (*a*) Finite differences simulation of a beamsplitter. (*b*) Top view SEM image of a splitting structure before wafer bonding. Scale bar denotes 1 μm. (*c*)–(*f*) SEM images of the exit side of beamsplitters with different spacings *S*. Scale bars denote 100 nm. (*g*) Schematic of the experimental geometry showing the coupling of the focused X-ray beam into the entrance of the beamsplitter, the subsequent guiding in the two channels, the free-space propagation behind the chip, and finally the far-field detector at a distance *D*. The far-field pattern shows the characteristic double-slit interference pattern, modulated with features of the waveguide modal structure. Arrows mark bifurcations in the interference fringes (fork-shaped structures). Length and angles are not to scale. (*h*) Enlarged view of the interference pattern with a sinusoidal fit to the intensity oscillations. (*i*) Scan in the *y* direction indicating the position of different beamsplitters which have all been defined on the same chip with different geometric parameters, and which can be selected by translating the chip in the FZP focus. Detailed scan profile of a single channel with a width (FWHM) of 282.6 nm giving an upper limit for the beam size in the horizontal direction.

Hoffmann-Urlaub, Salditt, Acta Cryst A, 2016
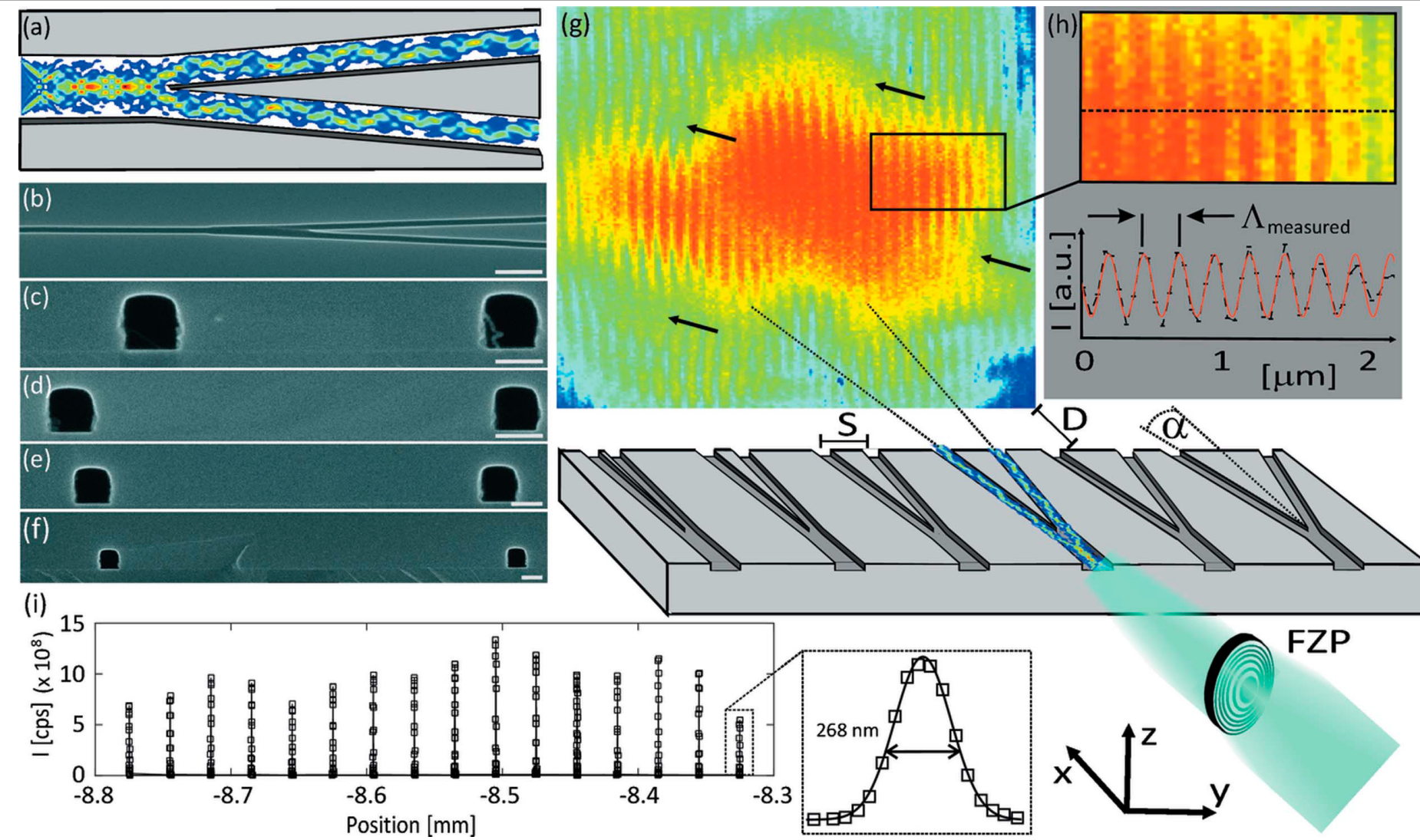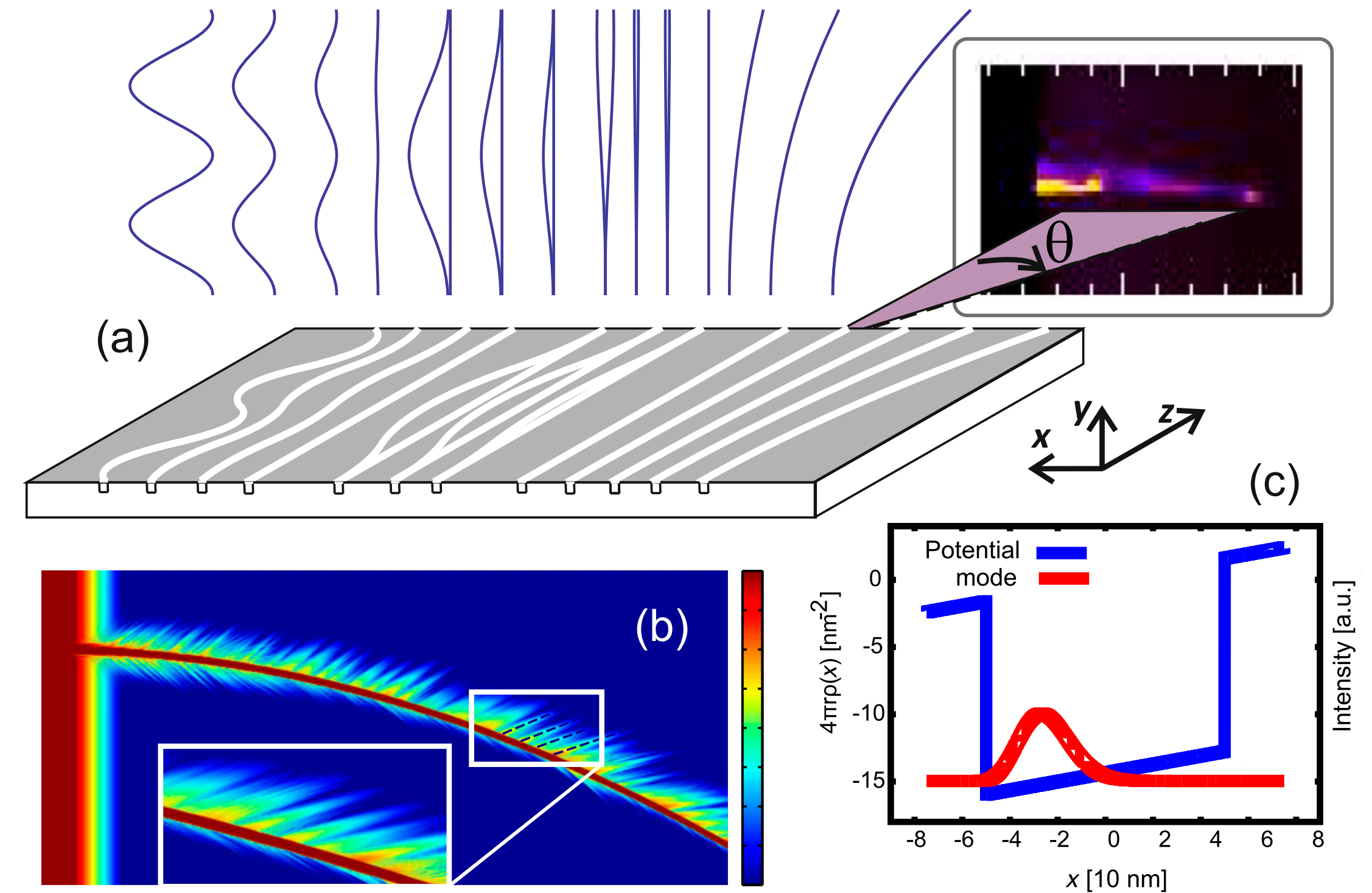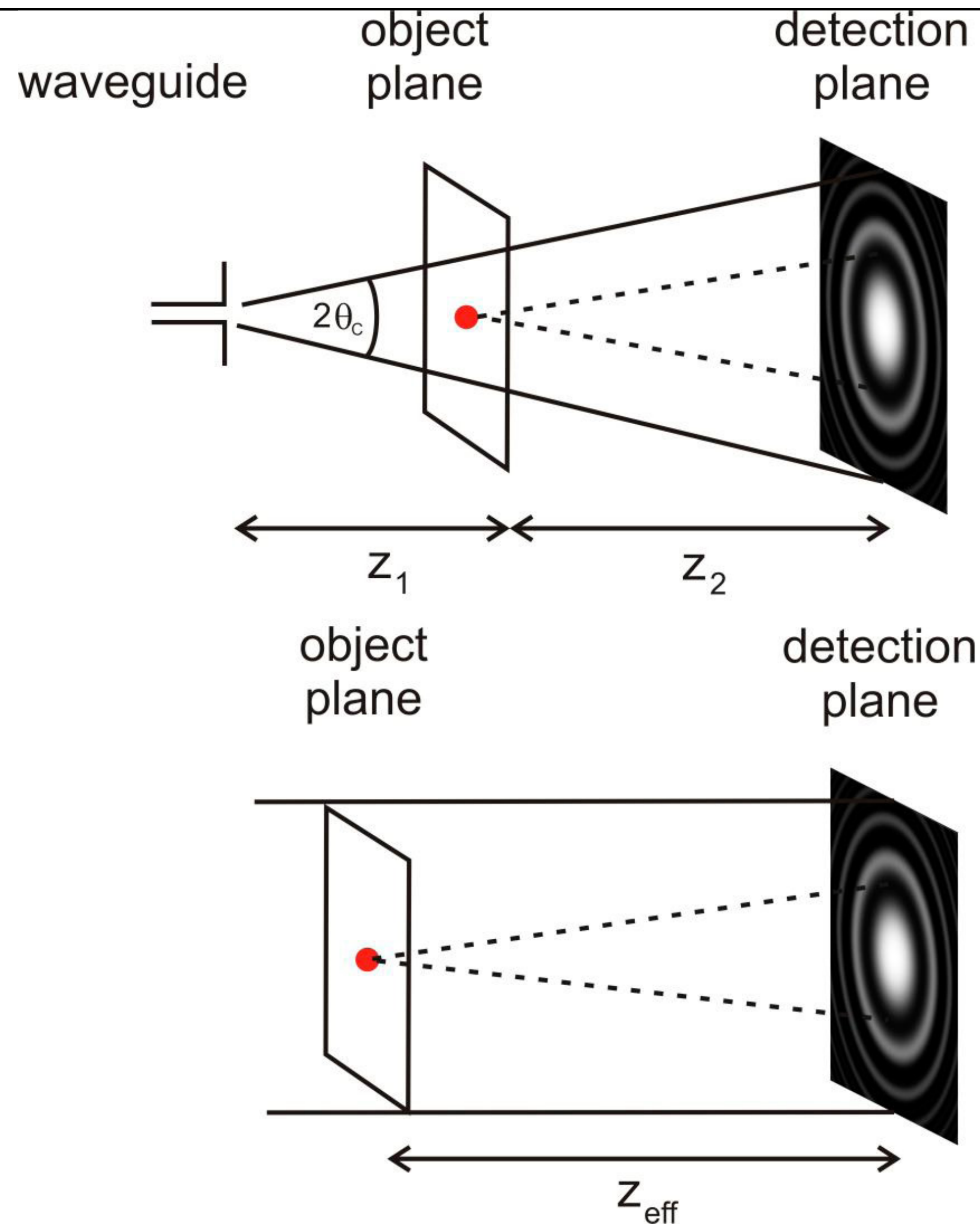
Tapering and Curving

Figure 1
Experimental geometry. (*a*) Finite differences simulation of a beamsplitter. (*b*) Top view SEM image of a splitting structure before wafer bonding. Scale bar denotes 1 μm. (*c*)–(*f*) SEM images of the exit side of beamsplitters with different spacings *S*. Scale bars denote 100 nm. (*g*) Schematic of the experimental geometry showing the coupling of the focused X-ray beam into the entrance of the beamsplitter, the subsequent guiding in the two channels, the free-space propagation behind the chip, and finally the far-field detector at a distance *D*. The far-field pattern shows the characteristic double-slit interference pattern, modulated with features of the waveguide modal structure. Arrows mark bifurcations in the interference fringes (fork-shaped structures). Length and angles are not to scale. (*h*) Enlarged view of the interference pattern with a sinusoidal fit to the intensity oscillations. (*i*) Scan in the *y* direction indicating the position of different beamsplitters which have all been defined on the same chip with different geometric parameters, and which can be selected by translating the chip in the FZP focus. Detailed scan profile of a single channel with a width (FWHM) of 282.6 nm giving an upper limit for the beam size in the horizontal direction.

Hoffmann-Urlaub, Salditt, Acta Cryst A, 2016

Salditt et al, PRL, 2015

waveguide object plane detection plane

$$M = \frac{z_1 + z_2}{z_1}$$

z_1 z_2

object plane detection plane

$$z_{eff} = \frac{z_1 \, z_2}{z_1 + z_2}$$

z_{eff}

Saldiṫ, Osterhoff et al, Journal of Synchrotron Radiation, 2015

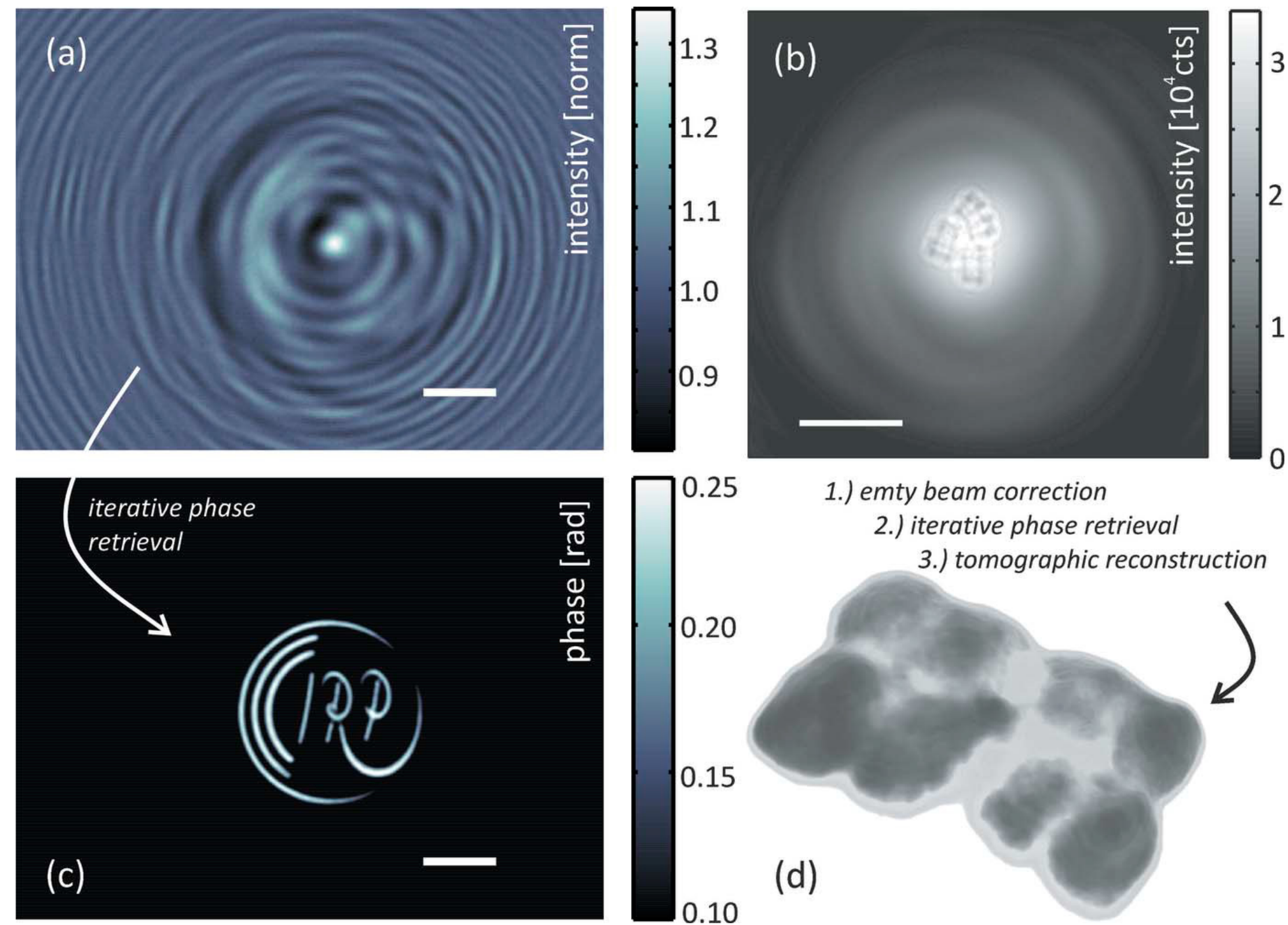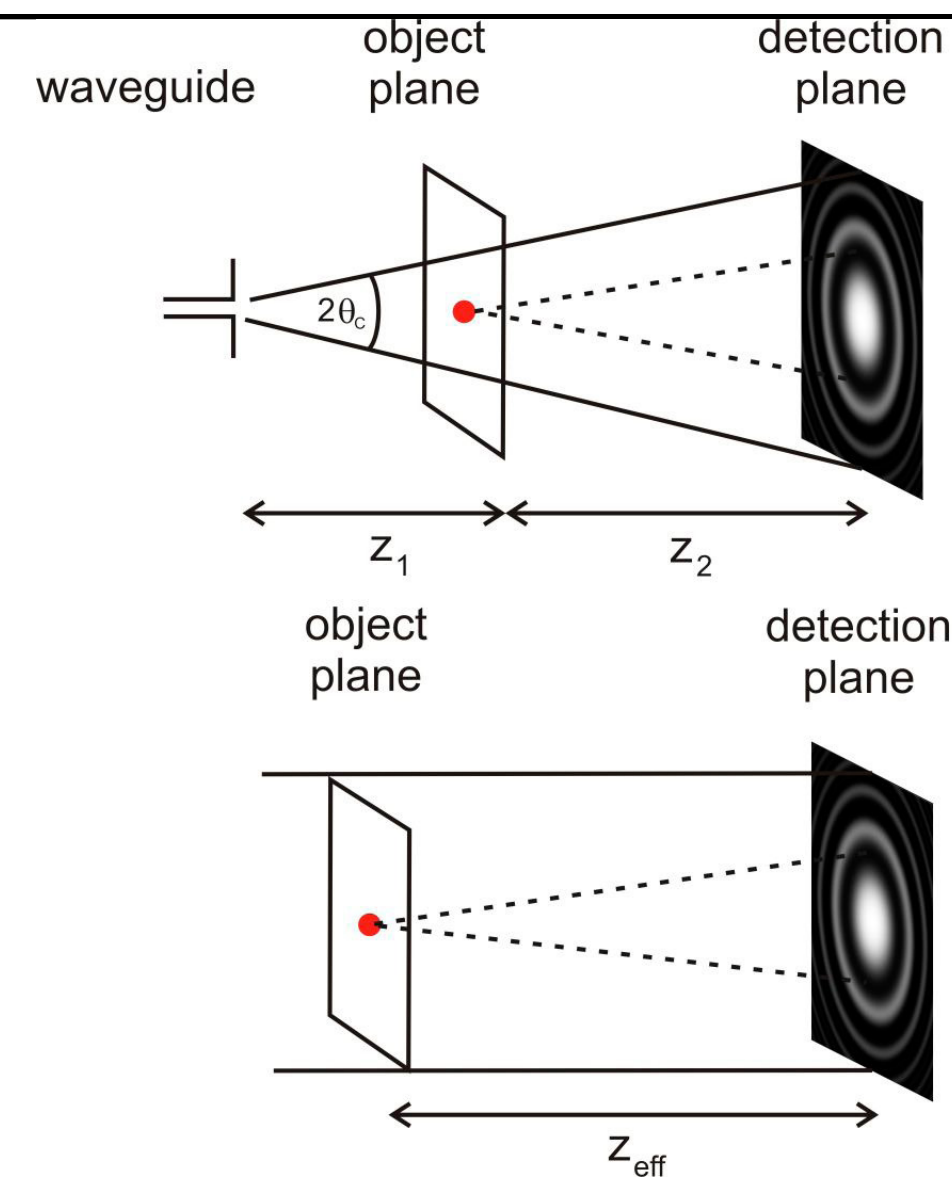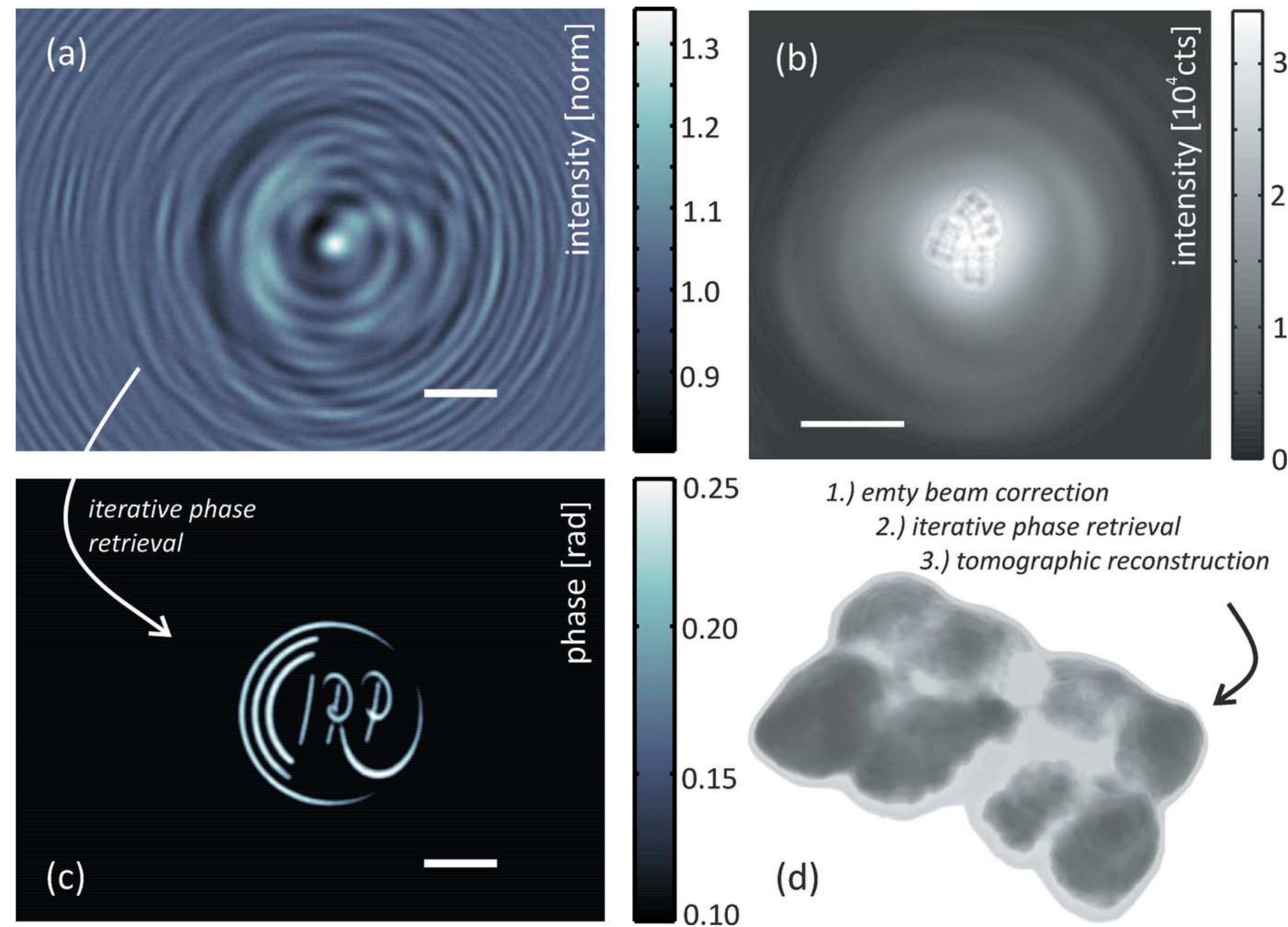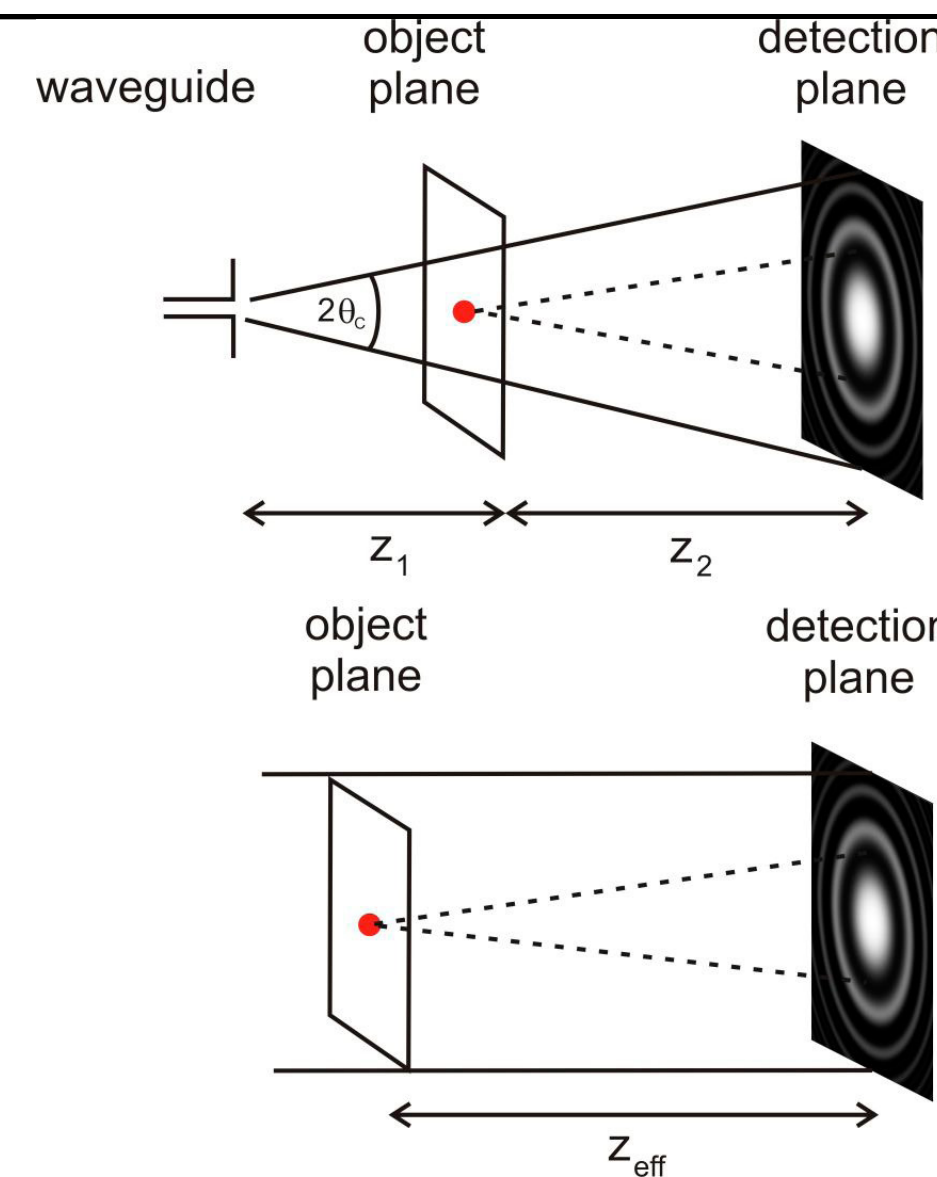**Figure 6**
Holographic imaging with the waveguide probe. Scale bar: 585 μm (detector plane). (*a*) Hologram of a test pattern after division by the empty beam. Scale bar: 6 mm (detector plane). (*b*) Raw data of a hologram with three *Deinococcus radiodurans* cells, showing the smooth line-shape and tails of the waveguide probe. (*c*) Phase reconstruction of the hologram shown in (*a*), with 22.4 nm pixel size. Scale bar: 2 μm. (*d*) Rendered three-dimensional density distribution of a *Deinococcus radiodurans* bacteria (Bartels *et al.*, 2012), reconstructed from waveguide-based holographic tomography.
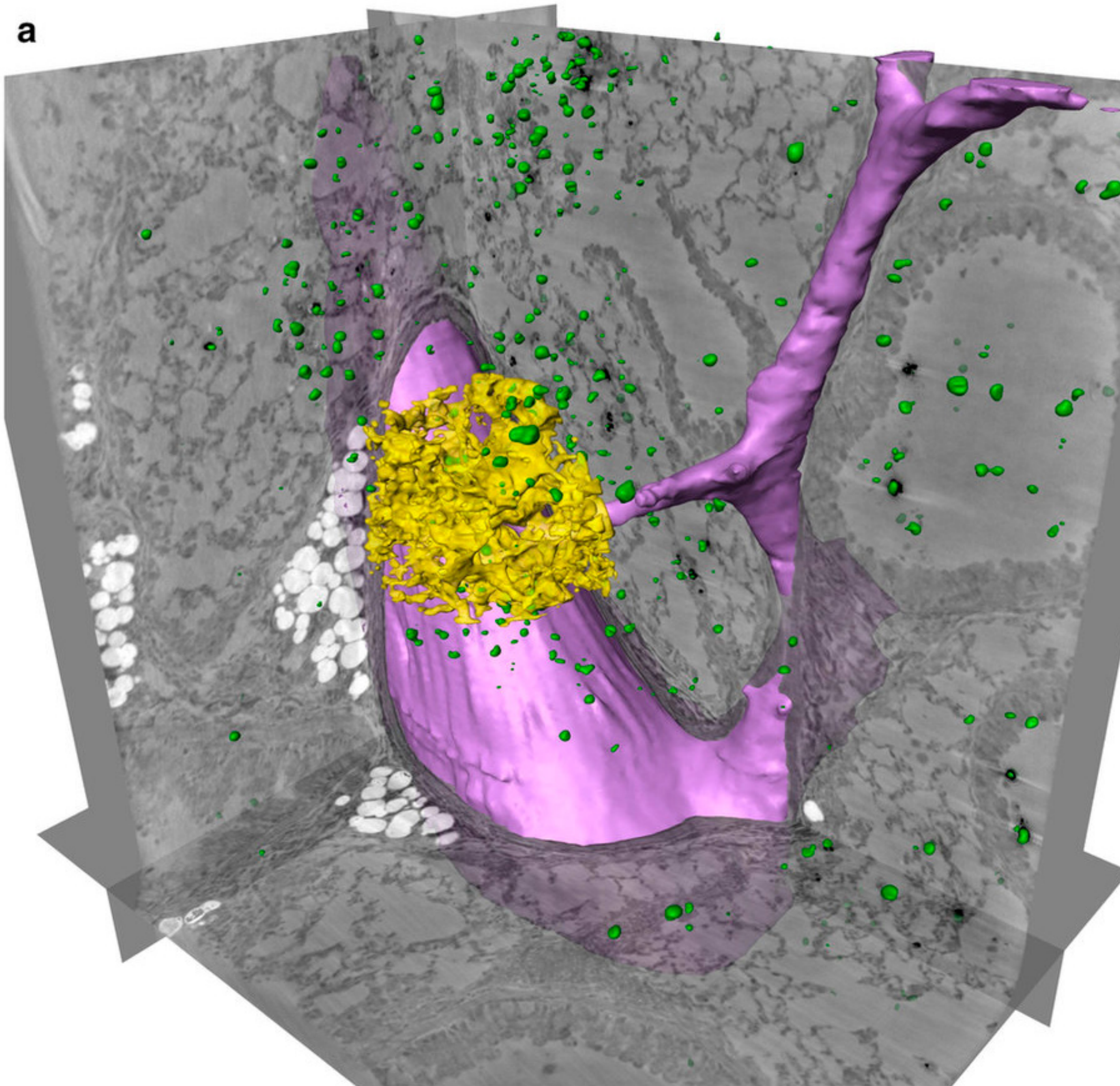
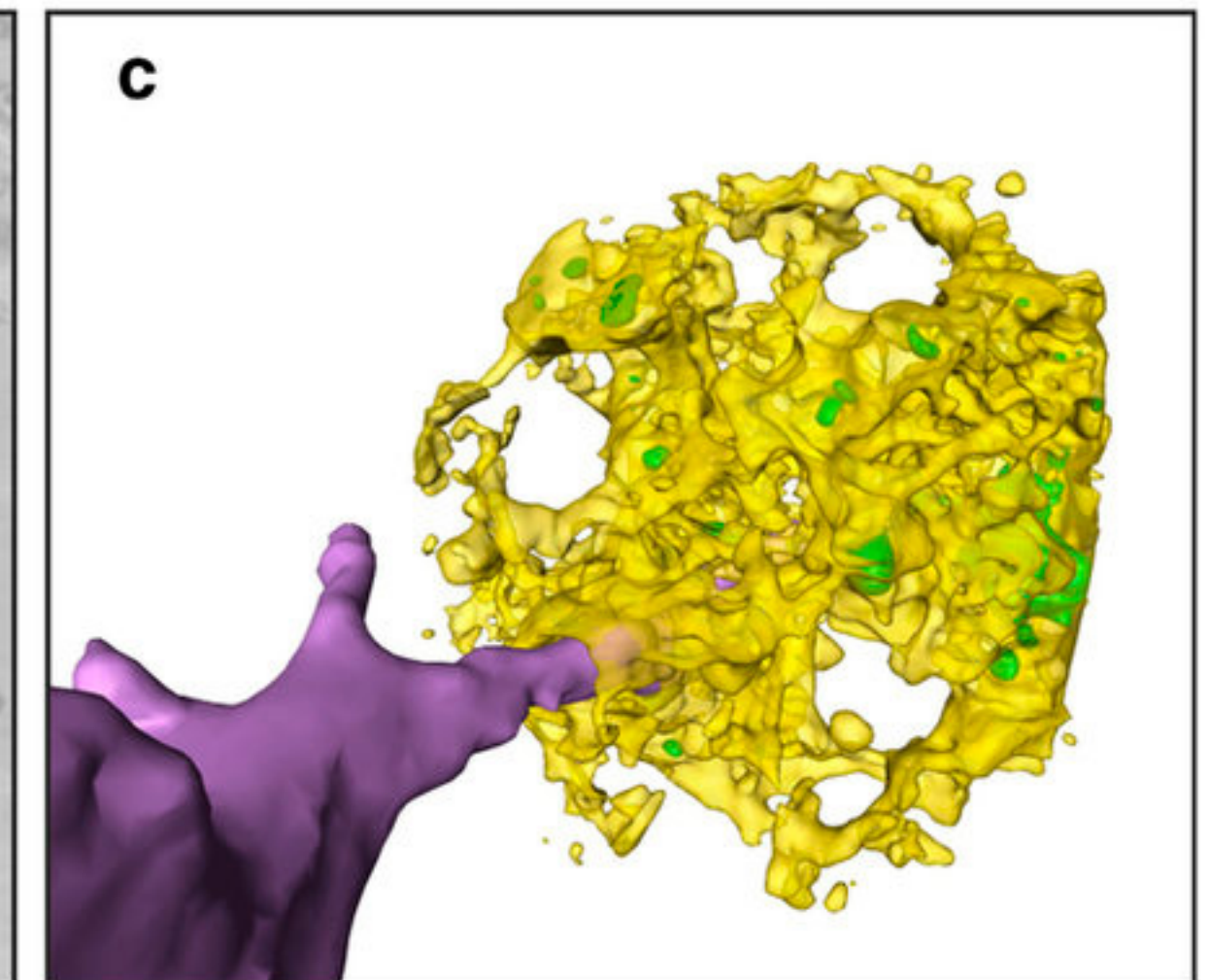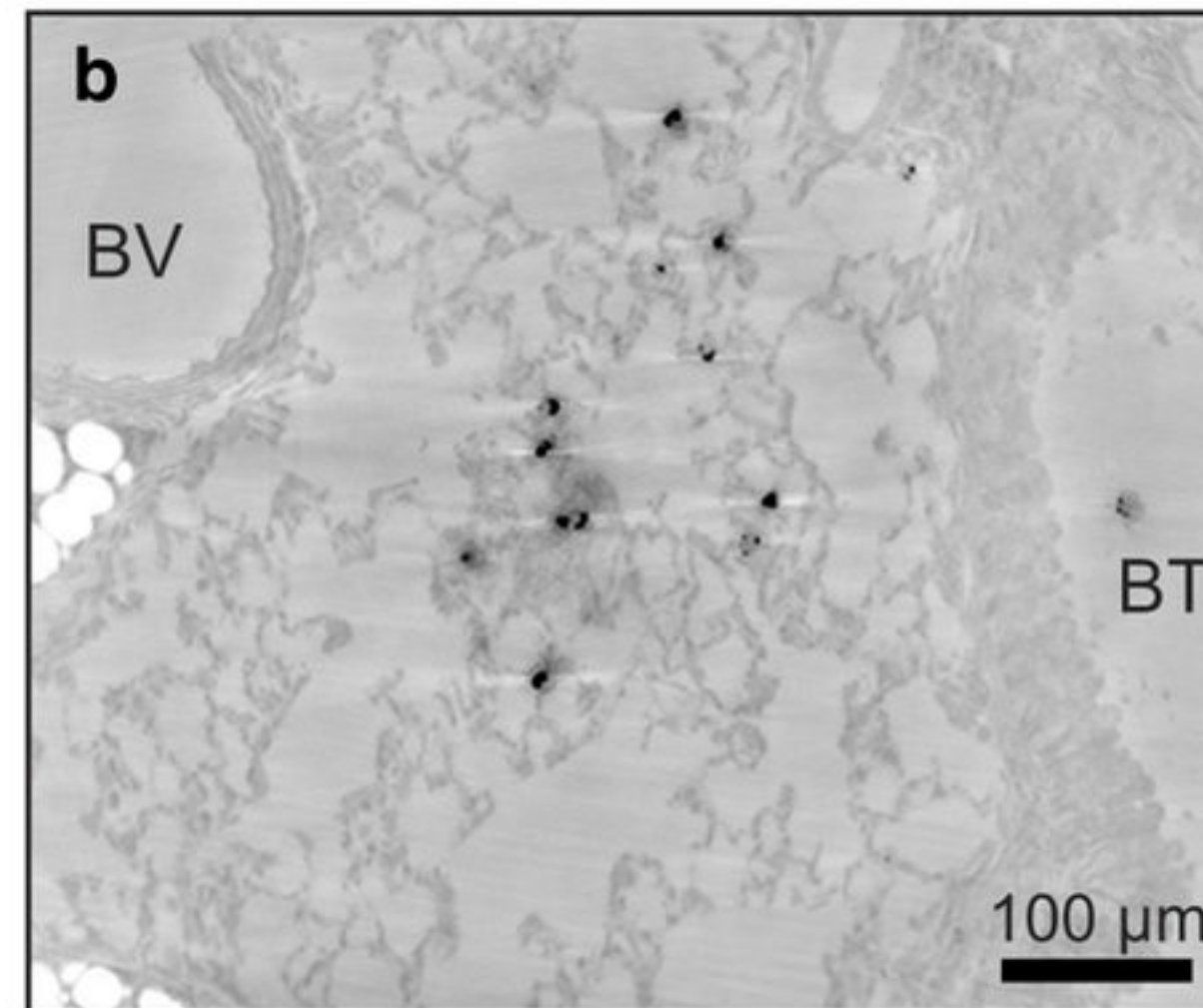$$M = \frac{z_1 + z_2}{z_1}$$

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Figure 6
Holographic imaging with the waveguide probe. Scale bar: 585 μm (detector plane). (*a*) Hologram of a test pattern after division by the empty beam. Scale bar: 6 mm (detector plane). (*b*) Raw data of a hologram with three *Deinococcus radiodurans* cells, showing the smooth line-shape and tails of the waveguide probe. (*c*) Phase reconstruction of the hologram shown in (*a*), with 22.4 nm pixel size. Scale bar: 2 μm. (*d*) Rendered three-dimensional density distribution of a *Deinococcus radiodurans* bacteria (Bartels *et al.*, 2012), reconstructed from waveguide-based holographic tomography.

$$M = \frac{z_1 + z_2}{z_1}$$

$$z_{eff} = \frac{z_1 \, z_2}{z_1 + z_2}$$

Advantages of WG-Holography

► full-field method, robust phase-retrieval

► coherence filtering, KB artefacts filtering

► geometrical magnification

► lower dose than ptychography

► compatible with tomography
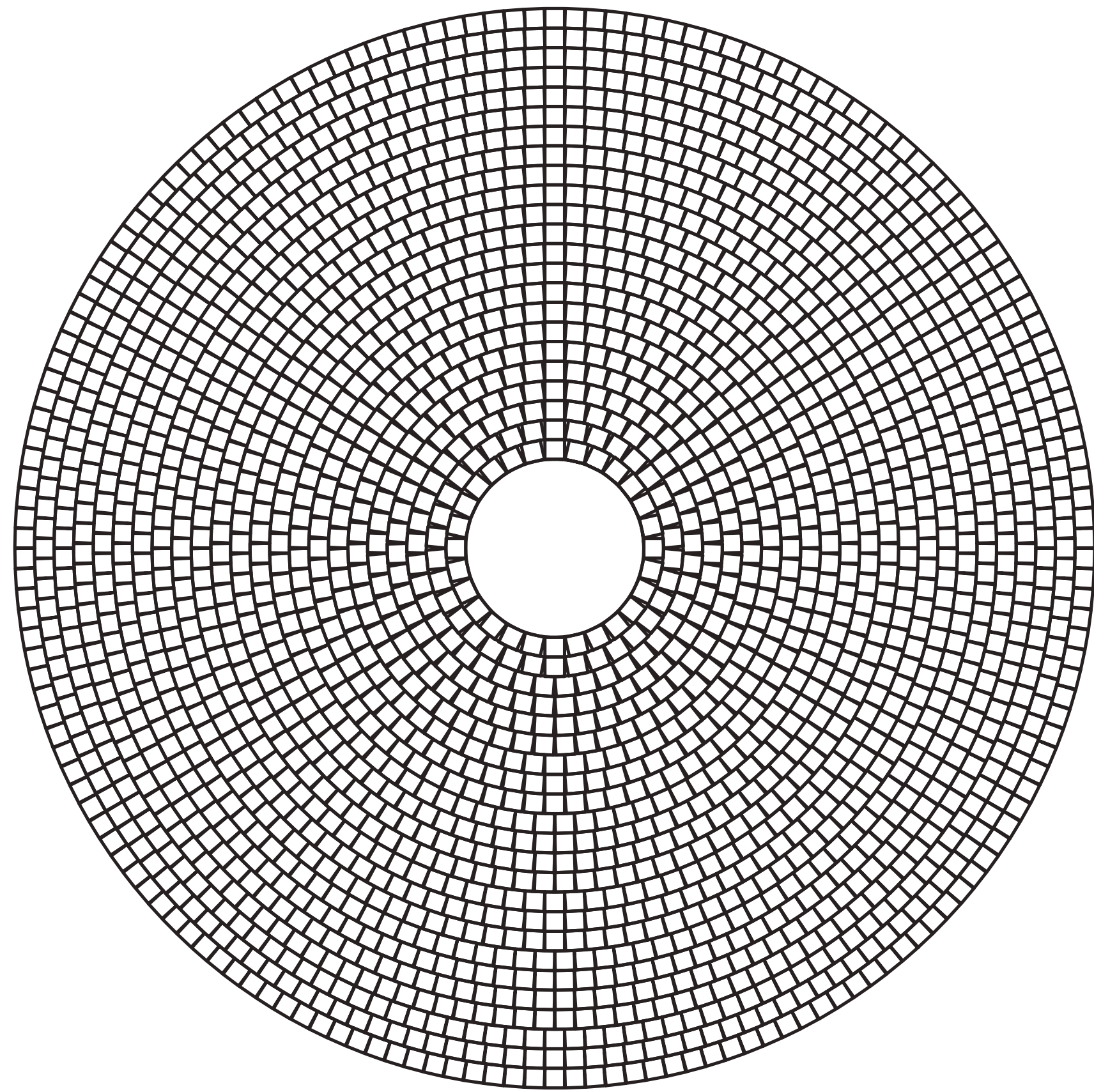
Salditt, Osterhoff et al, Journal of Synchrotron Radiation, 2015

- ▶ Reconstructed asthmatic lung tissue (mouse)
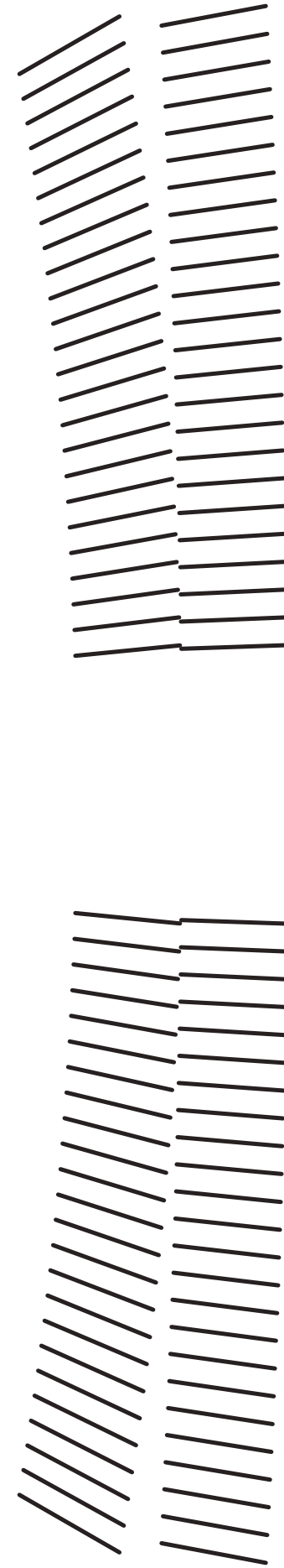- ▶ Sub-cellular resolution @ organic field of view
- ▶ Mass density in 3D!

Krenkel, Salditt et al, Scientific Reports, 2015

Discussion

(a)
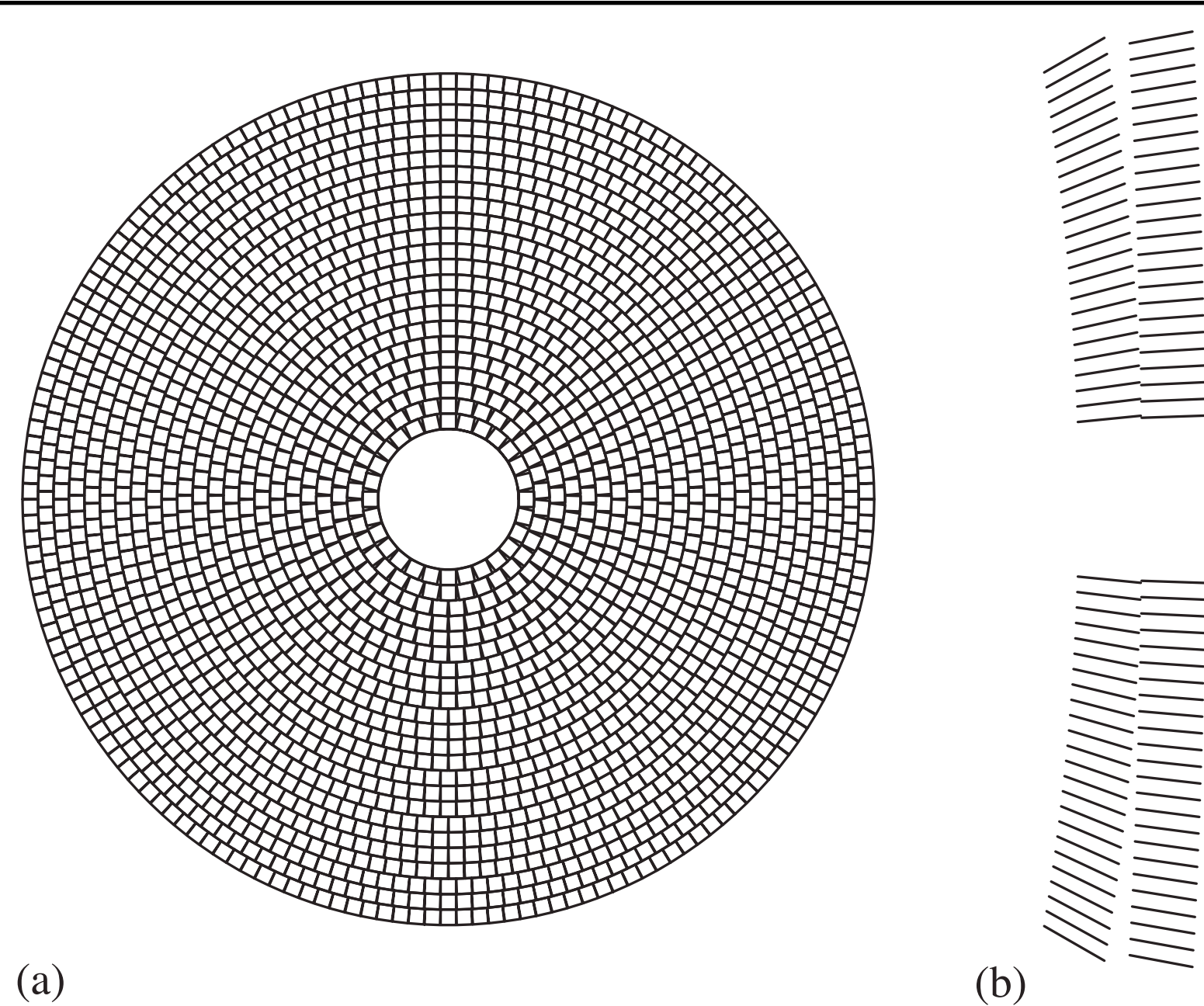
(b)

(a)

(b)

Fig. 1. (a) Packing arrangement around a solid core for a radially packed MCP composed of square multifibres. (b) Cross-section through Wolter pair.
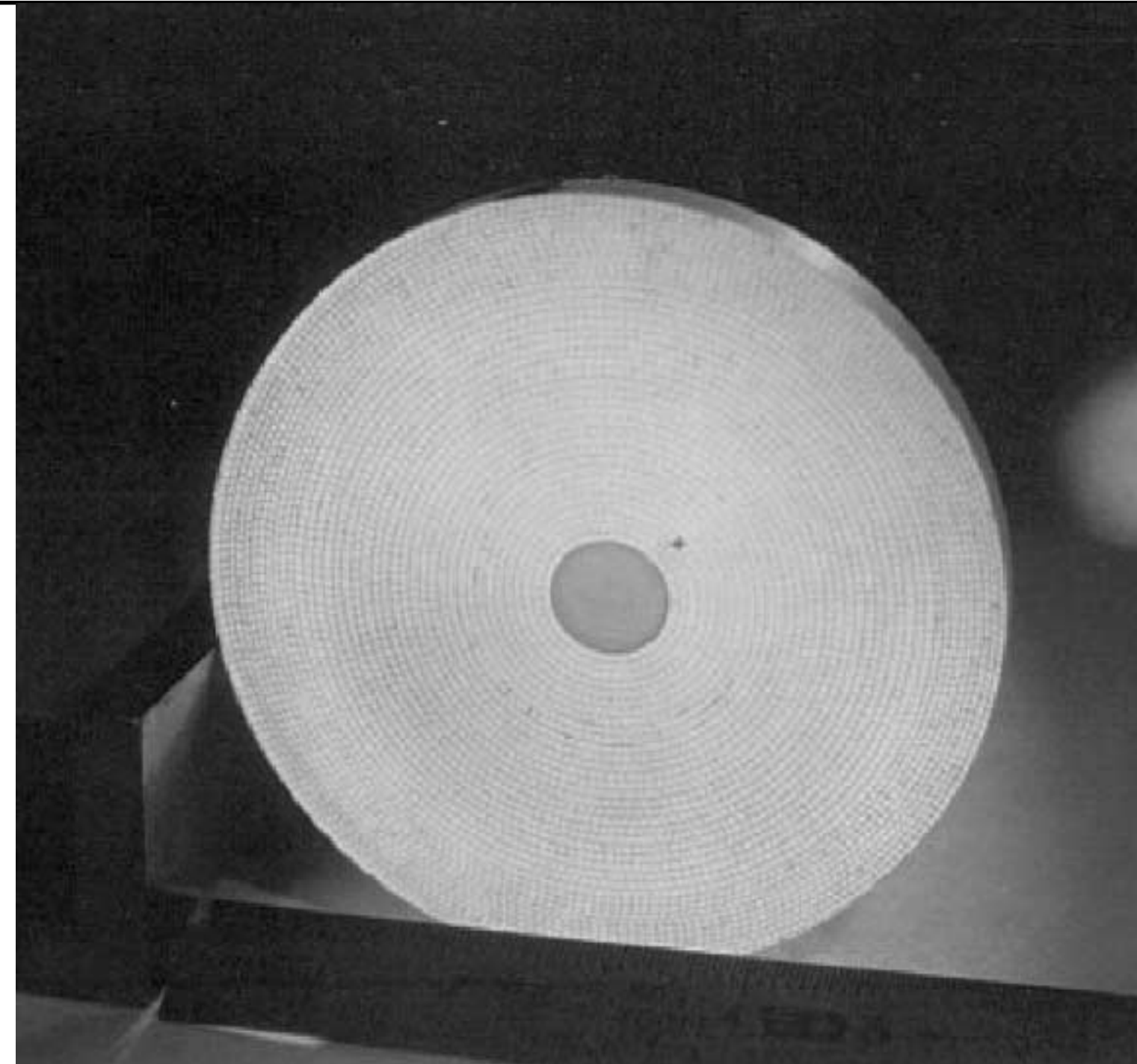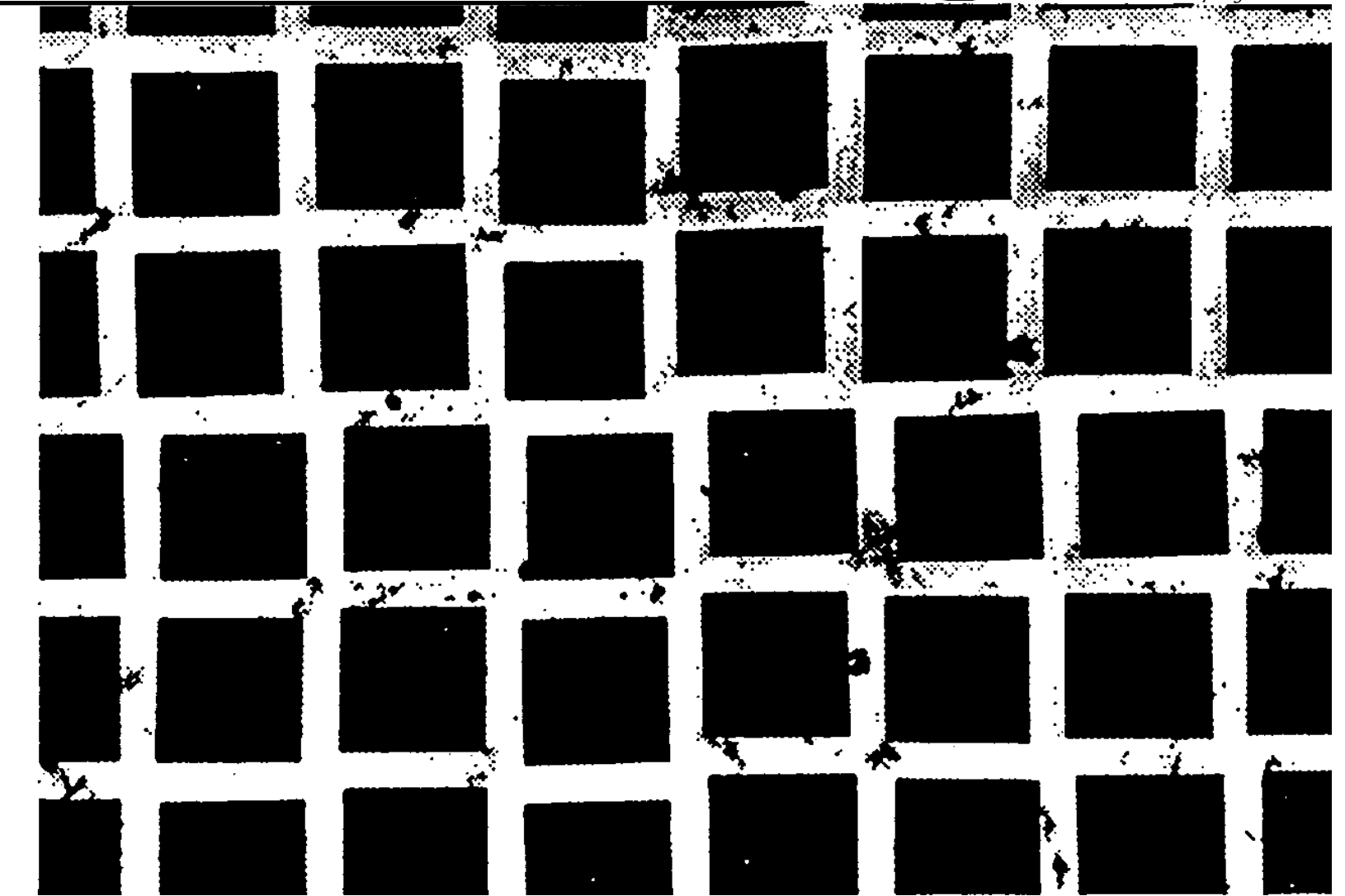

Fig. 2. Prototype radially packed MCP optic [28].

Fig. 2. Optical micrograph of our MCP.

Fig. 6. One-dimensional scan, along a focal arm, of intens

If it looks like a Zone Plate, it might be a Lobster eye :)

Wolter, Grundlagen der Optik, 1956
Price, Tomaselli et al, NIMA, 2002

Peele, Siegmund et al, Applied Optics, 1996

Summary and Wrap-Up

Mirrors

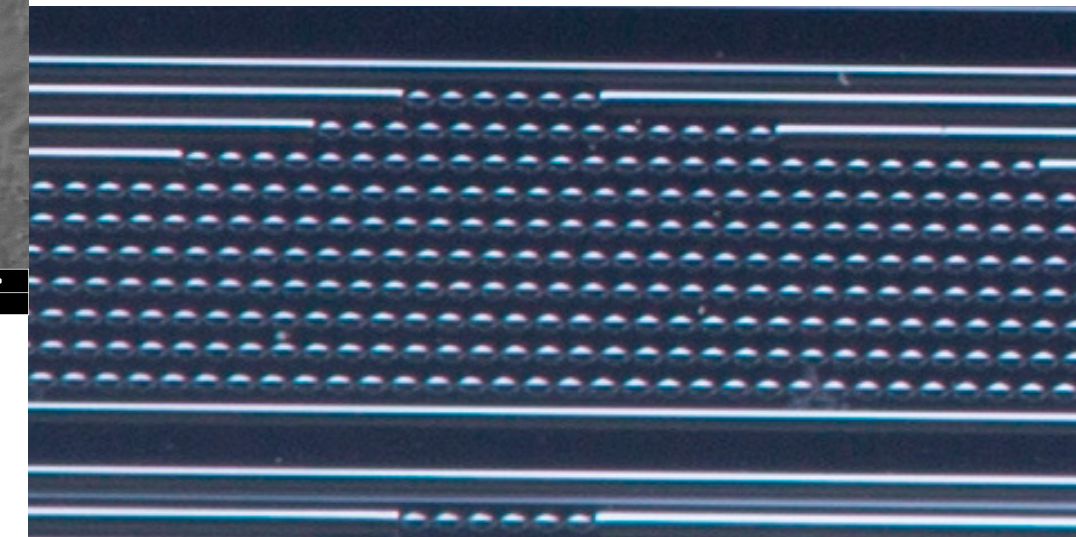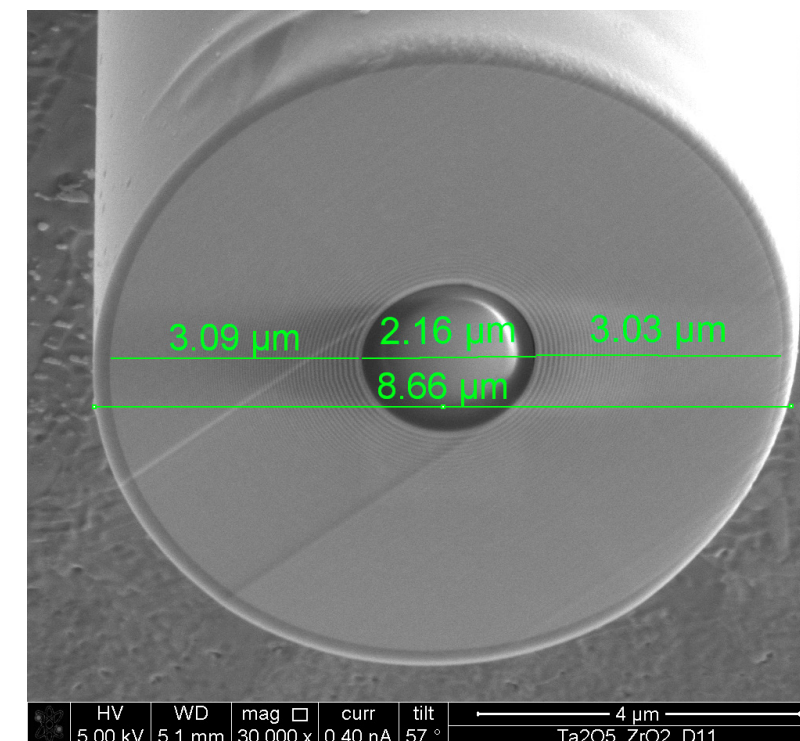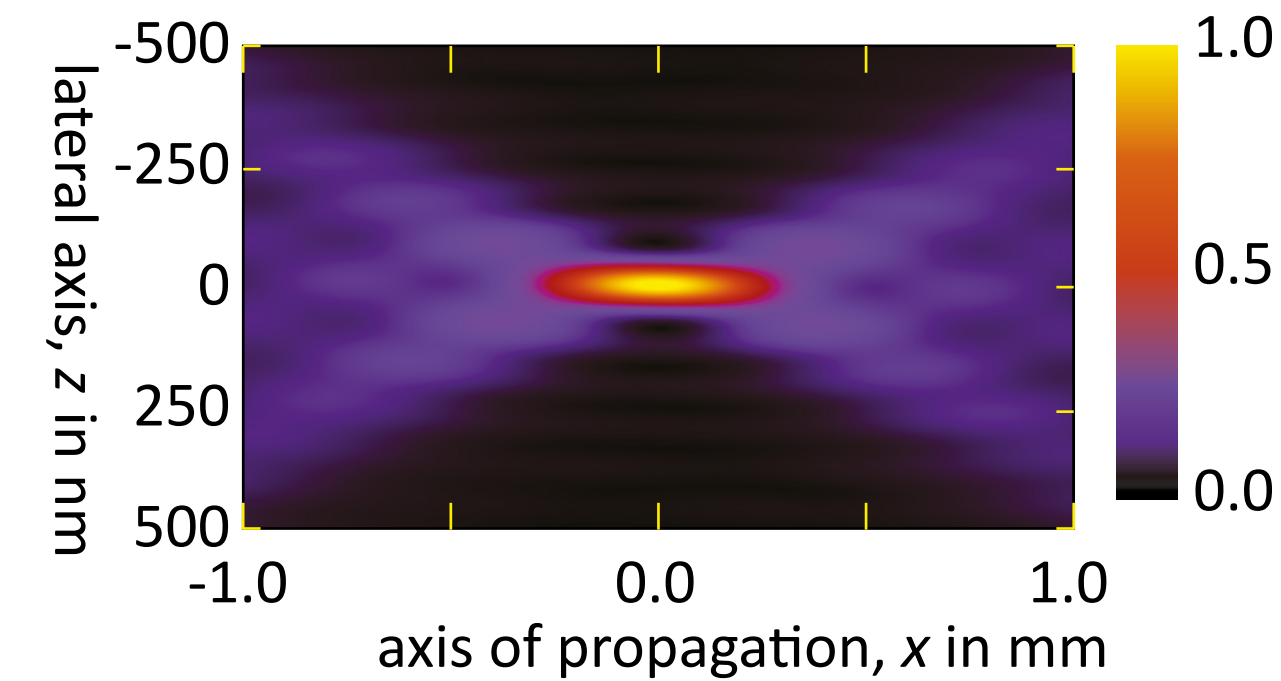► total reflection ≤ 17 keV

► multilayers ≥ 15 keV

Zone Plates

► Fresnel Zone Plates for soft x-rays

► Multilayer Laue Lenses, Multilayer Zone Plates for hard x-rays

► factor of 20 in energy = 20 years of work

Lenses

► possible, with 100+ lenses

ideal mirror

(a) Intensity distribution

Waveguides

► as coherence filter for holography

Outline

► Introduction + History

► Reflective Optics

► Diffractive Optics

► Refractive Optics

► Waveguides

► Discussion